Eric Pitman Summer Workshop in Computational Science

3. Descriptive Statistics

CENTER FOR COMPUTATIONAL RESEARCH

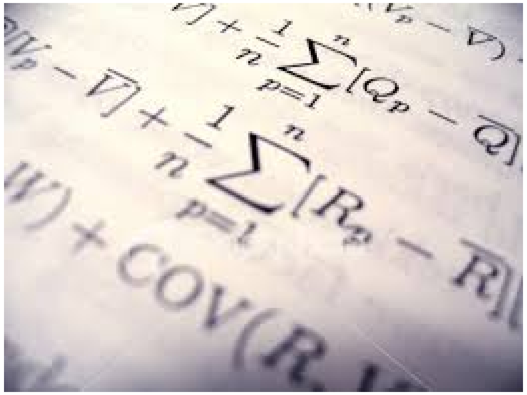
University at Buffalo
The State University of New York

Descriptive Statistics

Explore a dataset:

- What's in the dataset?
- What does it mean?
- What if there's *a lot* of it?

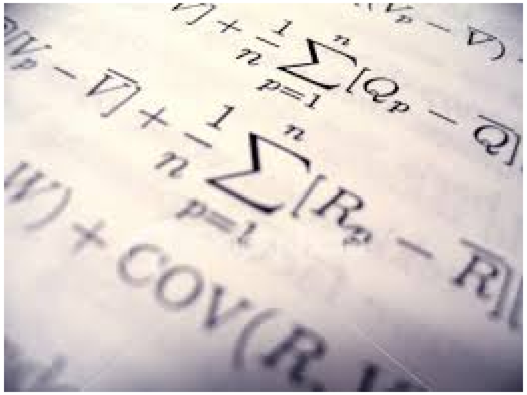
Basic Statistical Functions in R



Wanted: measures of the center and the spread of our numeric data.

- mean()
- median()
- range()
- var() and sd()   # variance, standard deviation
- summary()   # combination of measures

mean()



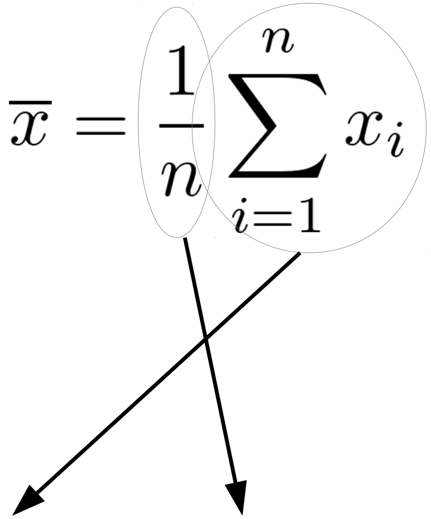
A measure of the data's “most typical” value.

- Arithmetic mean == average
- Divide sum of values by number of values

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

mean()

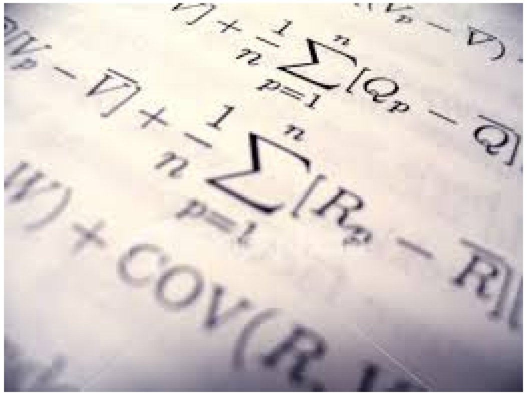
A measure of the data's “most typical” value.



$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

```
> f <- c(3, 2, 4, 1)
> mean(f)     # == sum(f)/length(f) == (3+2+4+1)/4
[1] 2.5
```

median()

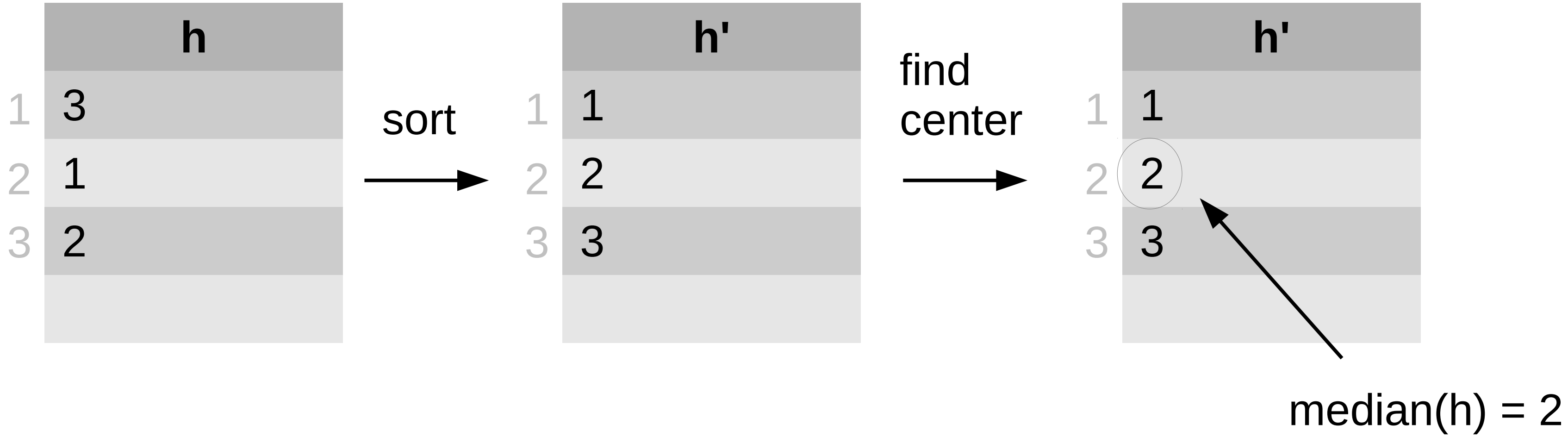


A measure of the data's center value. To find it:

- Sort the contents of the data structure
- Compute the value at the center of the data:
  - For odd number of elements, take the center element's value.
  - For even number of elements, take mean around center.

median()

Odd number of values:



	h
1	3
2	1
3	2

sort

	h'
1	1
2	2
3	3

find center

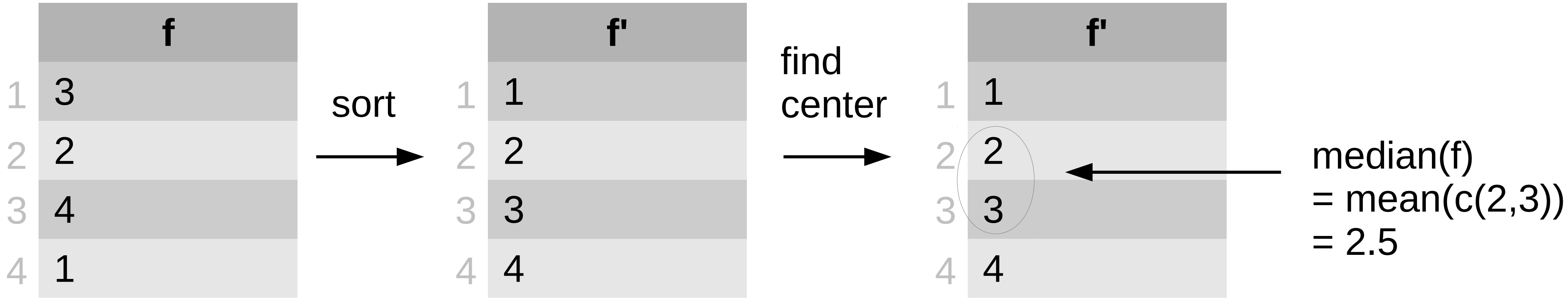
	h'
1	1
2	2
3	3

median(h) = 2

```
> h <- c(3, 1, 2)
> median(h)
[1] 2
```

median()

Even number of values: need to find mean()



	f
1	3
2	2
3	4
4	1

sort

	f'
1	1
2	2
3	3
4	4

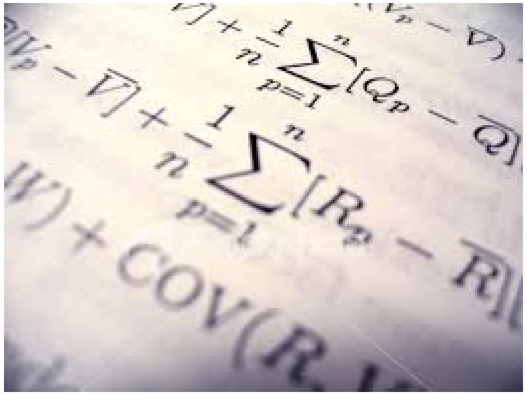
find center

	f'
1	1
2	2
3	3
4	4

median(f)
= mean(c(2,3))
= 2.5

```
> f <- c(3, 2, 4, 1)
> median(f)
[1] 2.50
```

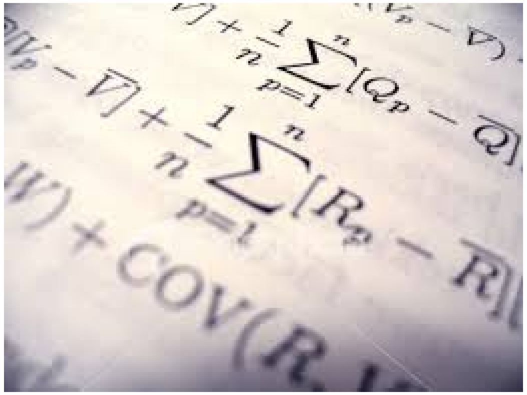
range():
min() and max()



range() reports the minimum and maximum values found in the data structure.

```
> f <- c(3, 2, 4, 1)
> range(f)   # reports min(f) and max(f)
  [1] 1   4
```

var() and sd()



- *Variance*: a measure of the spread of the values relative to their mean:

$$Var = s_n^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \overline{y})^2$$

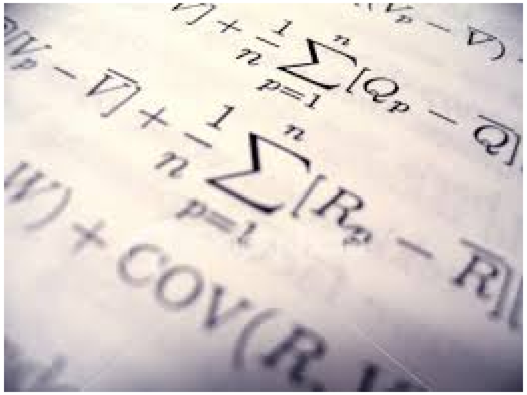
Sample variance

- *Standard deviation*: square root of the variance

$$s_n = \sqrt{Var}$$

Sample standard deviation

R's summary() Function

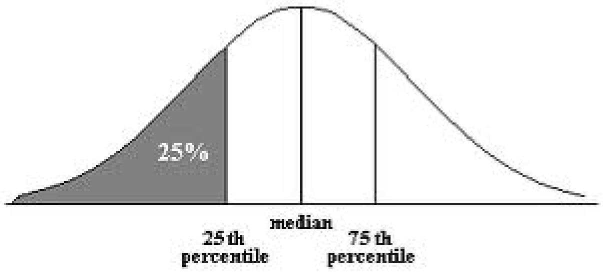


Provides several useful descriptive statistics about the data:

```
> g <- c(3, NA, 2, NA, 4, 1)
> summary(g)
  Min.    1st Qu.  Median   Mean      3rd Qu.   Max.    NA's
  1.00       1.75     2.50    2.50       3.25   4.00       2
```

Quartiles: Sort the data set and divide it up into quarters...

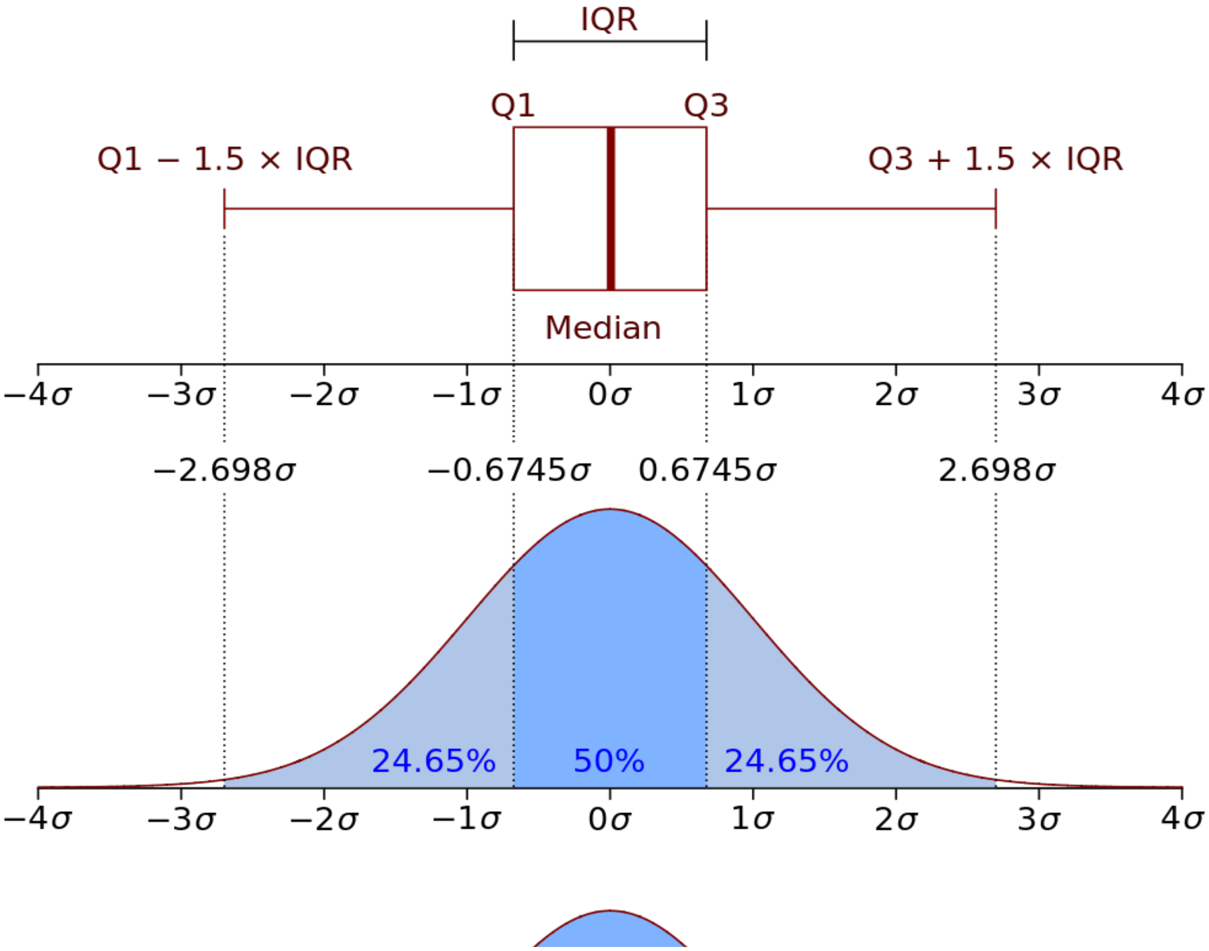
Quartiles



Quartiles are the *three points* that divide ordered data into four equal-sized groups:

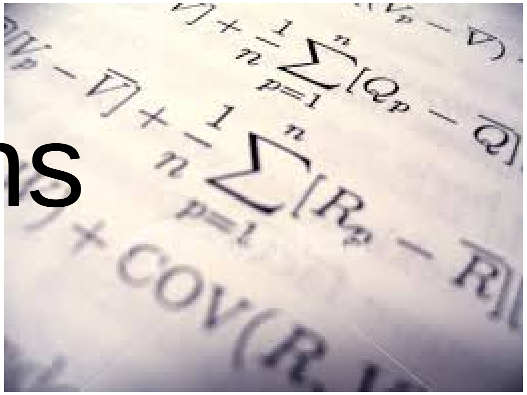
- Q1 marks the boundary just above the lowest 25% of the data
- Q2 (the *median*) cuts the data set in half
- Q3 marks the boundary just below the highest 25% of data

Quartiles



Boxplot and probability distribution function of Normal N(0,1σ^2) population

Summary: Basic Statistical Functions



- Characterize the center and the spread of our numeric data.
- Comparing these measures can give us a good sense of our dataset.

Statistics and Missing Data

If NAs are present, specify na.rm=TRUE to call:

- mean()
- median()
- range()
- sum()
- ...and some other functions

R disregards NAs, then proceeds with the calculation.

Diamonds Data

50,000 diamonds, for example:

	carat	cut	color	clarity	depth	table	price	x	y	z
1	0.23	Ideal	E	SI2	61.5	55	326	3.95	3.98	2.43
2	0.21	Premium	E	SI1	59.8	61	326	3.89	3.84	2.31
3	0.23	Good	E	VS1	56.9	65	327	4.05	4.07	2.31

What can we learn about these data?

Diamonds Data

This dataset is part of the ggplot2 library.

To enable access to the dataset, just load the library:

```
      > library("ggplot2")

     > str(diamonds)

'data.frame':53940 obs. of 10 variables:

 $ carat  : num  0.23 0.21 0.23 0.29
0.31  ...
```

Diamonds Data
summary()

Information provided by summary() depends on the type of data, by column:

```
    carat             cut              color       price
Min.   :0.2000   Fair     : 1610   D: 6775   Min.   :  326
1st Qu.:0.4000   Good     : 4906   E: 9797   1st Qu.:  950
Median :0.7000   Very Good:12082   F: 9542   Median : 2401
Mean   :0.7979   Premium  :13791   G:11292   Mean   : 3933
3rd Qu.:1.0400   Ideal    :21551   H: 8304   3rd Qu.: 5324
Max.   :5.0100                     I: 5422   Max.   :18823
                                   J: 2808
```

numeric data:
statistical summary

categorical (factor) data:
counts

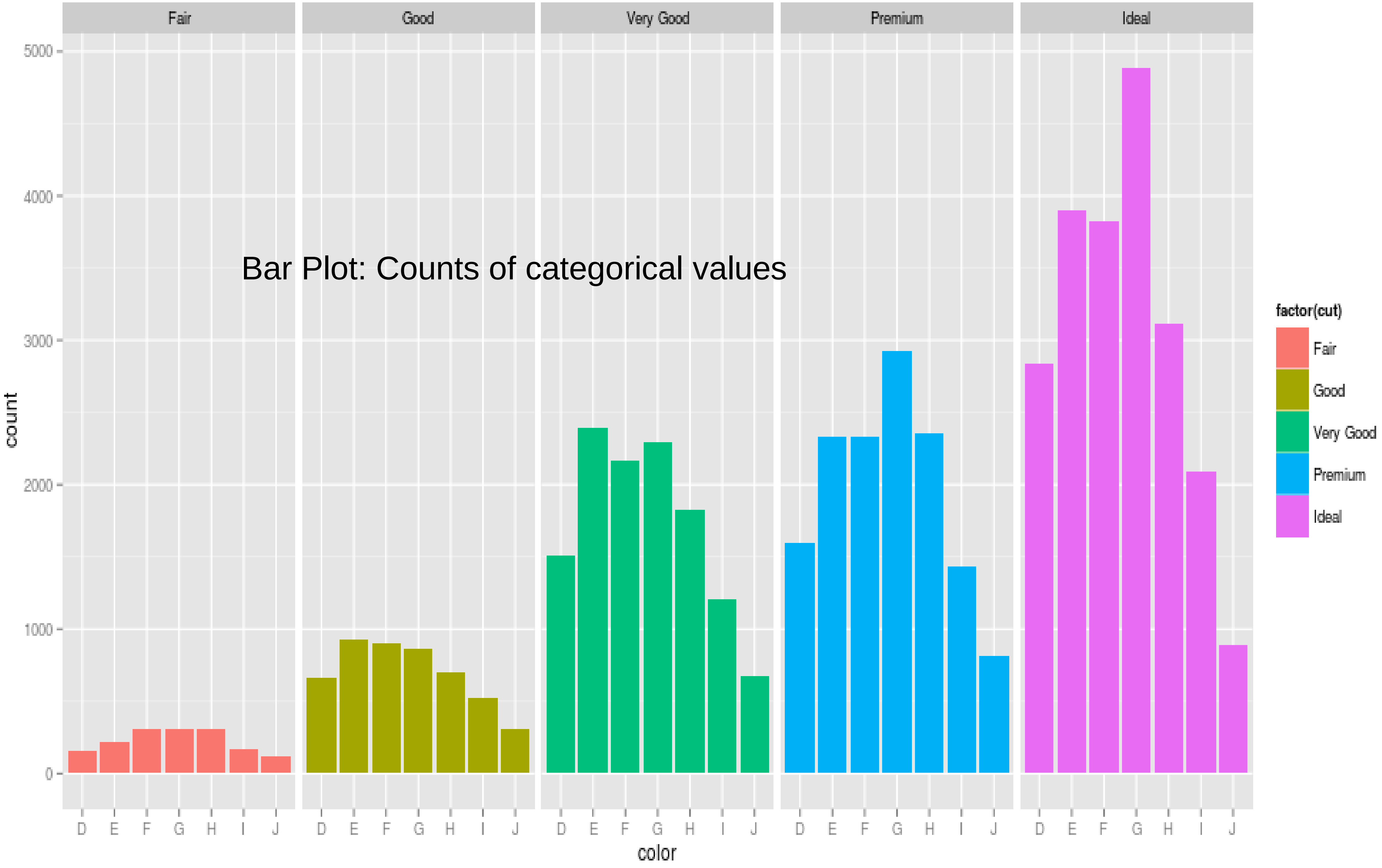
table() Function

Contingency table: counts of categorical values for selected columns

```
> table(diamonds$cut, diamonds$color)
```

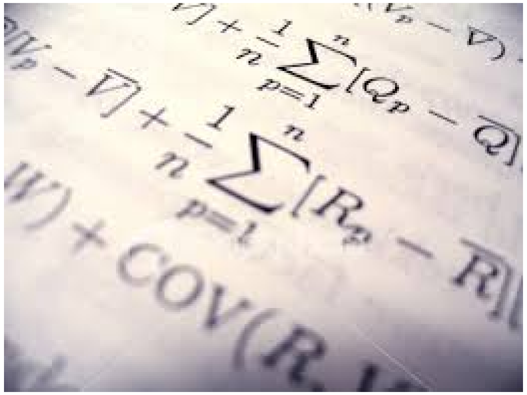
```
                D      E     F     G     H     I    J
  Fair        163    224   312   314   303   175  119
  Good        662    933   909   871   702   522  307
  Very Good  1513   2400  2164  2299  1824  1204  678
  Premium    1603   2337  2331  2924  2360  1428  808
  Ideal      2834   3903  3826  4884  3115  2093  896
```

Diamond Color and Cut



Bar Plot: Counts of categorical values

Correlation



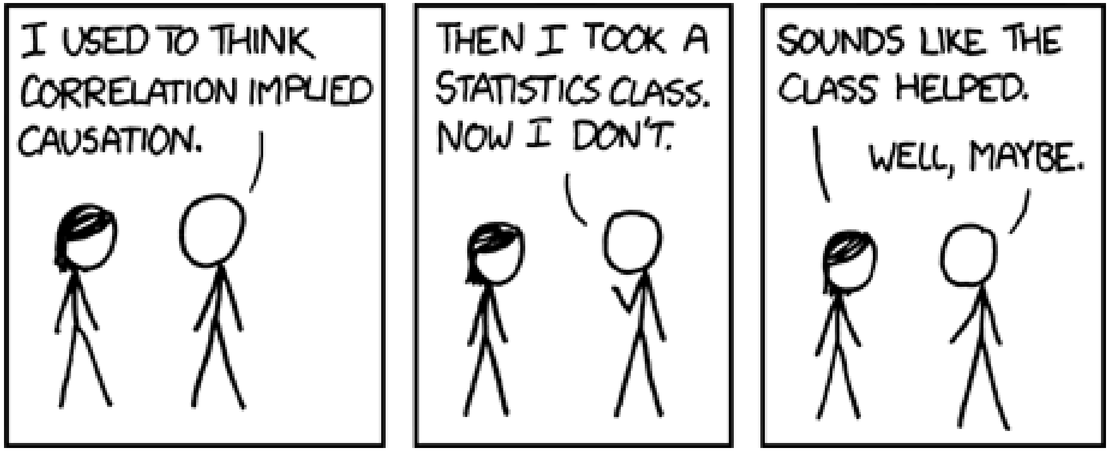
Do the two quantities X and Y vary together?

- Positively: $0 < \rho < 1$
- Or negatively: $-1 < \rho < 0$

$$\rho_{X,Y} = corr(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

A pairwise, *statistical* relationship between quantities

Correlation


I USED TO THINK CORRELATION IMPLIED CAUSATION.
THEN I TOOK A STATISTICS CLASS. NOW I DON'T.
SOUNDS LIKE THE CLASS HELPED.
WELL, MAYBE.

$$\rho_{X,Y} = corr(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

NOTE: Correlation does not imply causation...

Looking for Correlations

diamonds data frame: 50,000 diamonds

- carat: weight of the diamond (0.2–5.01)
- table: width of top of diamond relative to widest point (43–95)
- price: price in US dollars
- x: length in mm (0–10.74)
- y: width in mm (0–58.9)
- z: depth in mm (0–31.8)

cor() Function

Look at pairwise, *statistical* relationships between numeric data:

```
> cor(diamonds[c(1,6:10)])
            carat     table     price         x         y         z
carat   1.0000000 0.1816175 0.9215913 0.9750942 0.9517222 0.9533874
table   0.1816175 1.0000000 0.1271339 0.1953443 0.1837601 0.1509287
price   0.9215913 0.1271339 1.0000000 0.8844352 0.8654209 0.8612494
x       0.9750942 0.1953443 0.8844352 1.0000000 0.9747015 0.9707718
y       0.9517222 0.1837601 0.8654209 0.9747015 1.0000000 0.9520057
z       0.9533874 0.1509287 0.8612494 0.9707718 0.9520057 1.0000000
```

-1.0: perfectly anticorrelated

↕

0 : uncorrelated

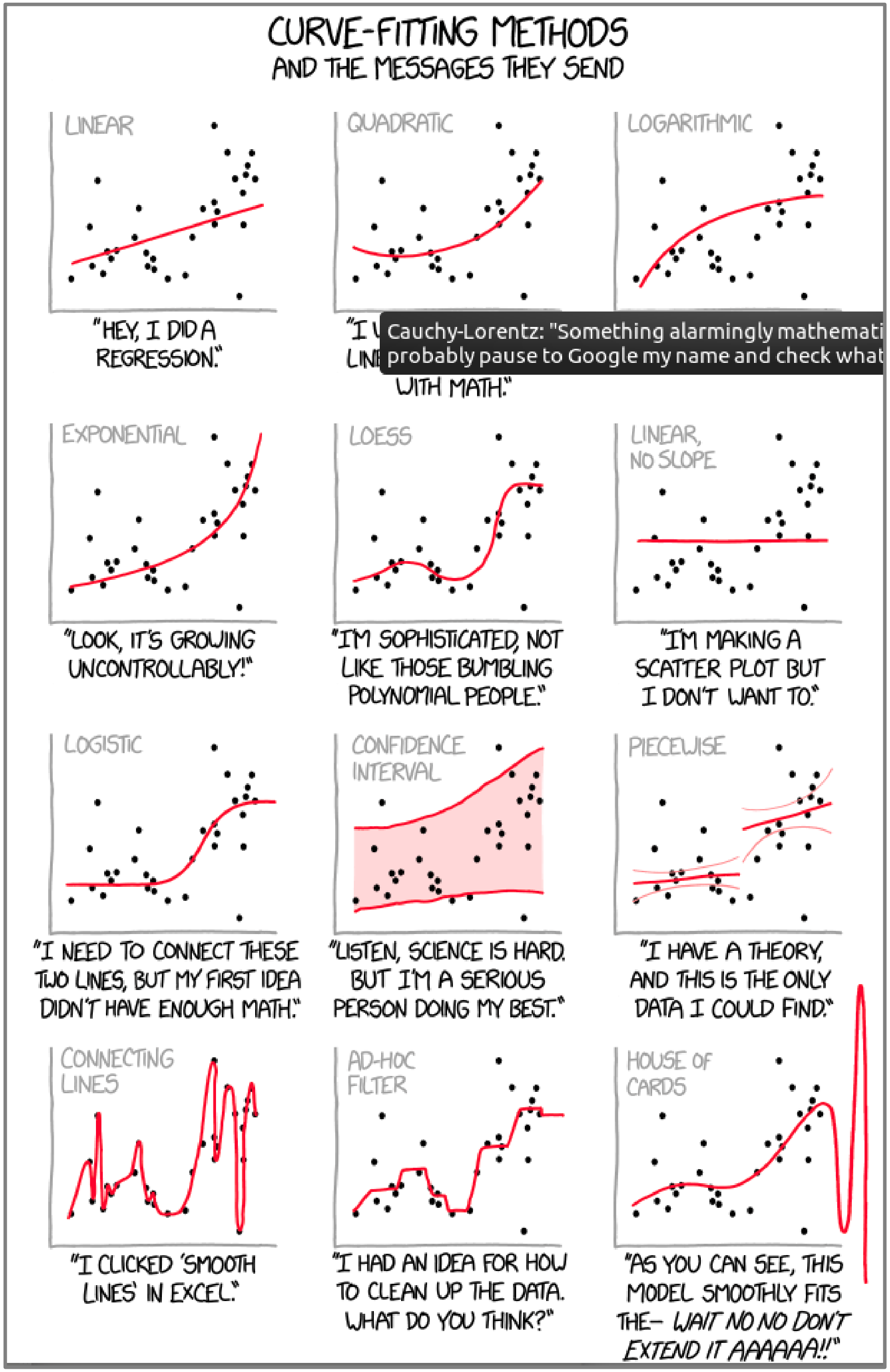
↕

1.0: perfectly correlated

cor() on categorical data

```
cut=as.integer(diamonds$cut)

color=as.integer(diamonds$color)

clarity=as.integer(diamonds$clarity)

compare=data.frame(diamonds$price, cut, color, clarity)

cor(compare)
```

```
                 diamonds.price         cut       color     clarity
diamonds.price     1.00000000 -0.05349066  0.17251093 -0.14680007
cut               -0.05349066  1.00000000 -0.02051852  0.18917474
color              0.17251093 -0.02051852  1.00000000  0.02563128
clarity           -0.14680007  0.18917474  0.02563128  1.00000000
```


CURVE-FITTING METHODS
AND THE MESSAGES THEY SEND
LINEAR
"HEY, I DID A REGRESSION."
QUADRATIC
"I W
LINE
WITH MATH."
LOGARITHMIC
EXPONENTIAL
"LOOK, IT'S GROWING UNCONTROLLABLY!"
LOESS
"I'M SOPHISTICATED, NOT LIKE THOSE BUMBLING POLYNOMIAL PEOPLE."
LINEAR, NO SLOPE
"I'M MAKING A SCATTER PLOT BUT I DON'T WANT TO."
LOGISTIC
"I NEED TO CONNECT THESE TWO LINES, BUT MY FIRST IDEA DIDN'T HAVE ENOUGH MATH."
CONFIDENCE INTERVAL
"LISTEN, SCIENCE IS HARD. BUT I'M A SERIOUS PERSON DOING MY BEST."
PIECEWISE
"I HAVE A THEORY, AND THIS IS THE ONLY DATA I COULD FIND."
CONNECTING LINES
"I CLICKED 'SMOOTH LINES' IN EXCEL."
AD-HOC FILTER
"I HAD AN IDEA FOR HOW TO CLEAN UP THE DATA. WHAT DO YOU THINK?"
HOUSE OF CARDS
"AS YOU CAN SEE, THIS MODEL SMOOTHLY FITS THE- WAIT NO NO DON'T EXTEND IT AAAAAA!!"

Cauchy-Lorentz: "Something alarmingly mathemati
probably pause to Google my name and check what

Be careful!

https://xkcd.com/2048/

https://www.explainxkcd.com/wiki/index.php

Student Dataset Example

Now we can write some R to perform some descriptive statistics on our student data:

- Contingency table of school and handedness?
- Correlation between age and height?
- Some descriptive statistics about height (only comparing apples to apples!)

OK...but what's the problem here? Small dataset!

Interlude

Complete descriptive statistics exercises.

Open in the RStudio source editor:

<workshop>/exercises/3-exercises-descriptive-statistics.R