# Sentence, Proposition, Judgment, Statement, and Fact: Speaking about the Written English Used in Logic

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Dedication: *for Professor Newton da Costa, friend, collaborator, co-discoverer of the Truth-set Principle, and co-creator of the Classical Logic of Variable-binding Term Operators-on his eightieth birthday.*

ABSTRACT. The five ambiguous words—sentence, proposition, judgment, statement, and fact—each have meanings that are vague in the sense of admitting borderline cases. This paper discusses several senses of these and related words used in logic. It focuses on a constellation of recommended primary senses. A *judgment* is a *private* epistemic act that results in a new belief; a *statement* is a *public* pragmatic event involving an utterance. Each is executed by a unique person at a unique time and place. Propositions and sentences are timeless and placeless abstractions. A *proposition* is an intensional entity; it is a meaning composed of concepts. A *sentence* is a linguistic entity. A written sentence is a string of characters. A sentence can be used by a person to express meanings, but no sentence is intrinsically meaningful. Only propositions are properly said to be true or to be false—in virtue of *facts*, which are subsystems of the universe. The fact that two is even is timeless; the fact that Socrates was murdered is semi-eternal; the most general facts of physics—in virtue of which propositions of physics are true or false—are eternal. As suggested by the title, this paper is meant to be read aloud.

## 1 Introduction

The words—sentence, proposition, judgment, statement, and fact—are *ambiguous* in that logicians use each of them with multiple normal meanings. Several of their meanings are *vague* in the sense of admitting borderline cases. This paper juxtaposes, distinguishes, and analyzes several senses of

these and related words, focusing on a constellation of recommended senses. According to the recommendation, a *judgment* is a private epistemic event that results in a new belief and a *statement* is a public pragmatic event, an act of writing or speaking. Both are made by a unique person at a unique time and place. Judgments are often not voluntary, although they cannot be coerced. Statements are usually voluntary, although they can be coerced. In contrast, propositions and sentences are timeless and placeless abstractions. A *proposition* is an intensional entity; in some cases it is a meaning of a sentence: it is a meaning composed of concepts, a complex sense composed of simpler senses. A [declarative] *sentence* is a linguistic entity. A written sentence is a string of characters; it is composed of character-strings—usually words or "symbols"[1] that can be used to express meanings, but which are not in themselves meaningful. A spoken sentence is composed of [articulate] sounds. Only a proposition is properly said to be true or to be false—although in certain contexts, or with suitable qualifications, judgments, statements, or even sentences, may be said to be true or false in appropriate derivative senses. Propositions are true or false in virtue of *facts*, which are either timeless or temporal subsystems of the universe.[2]

A fact is *timeless* if it is only about timeless entities such as numbers. The fact that two is even is timeless. The proposition that two is even can only be expressed using the *timeless-present* tense.[3] It is incoherent to say that two is presently even or that two is still even. A fact is *temporal* if it is about temporal entities such as material objects. Temporal facts are semi-eternal or eternal. A semi-eternal fact comes into being in an interval of time and it persists eternally thereafter. The fact that Socrates was murdered can never be erased. An eternal fact exists without having come into being. The most general facts of physics—in virtue of which propositions of physics are true or false—are eternal. In the sense intended here, a fact never changes. Facts are prior to propositions in the same sense that the past is prior to

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[1]'Character', 'sign', and 'symbol' are often exact synonyms in logic usage. Many logicians show no awareness of the awkwardness of the fact that, in the senses they prefer, characters do not characterize, signs do not signify, and symbols do not symbolize—except under an interpretation, i.e., extrinsically, never intrinsically. See Corcoran, Frank, and Maloney *1974*.

[2]The proposition that Plato taught Aristotle, which is composed of concepts or senses, is true in virtue of a fact composed of historical entities that are not concepts. The fact in question is the fact that Plato taught Aristotle, which has Plato and Aristotle as constituents. The word 'fact' has been used in many other senses. For example, Frege (*1918/1956*, 307) takes facts to be true propositions, in a sense of 'proposition' very close to the one recommended in the current article. Later in the same article on page 311, he uses the word 'fact' in yet another sense: he says that thinking, judging, understanding, and the like are "facts of human life". Austin (*1961*, 91) disapproves of taking 'fact' as synonymous with 'true statement'.

[3]Frege made a similar point (*1918/1956*, 309–310).

historical truths and nature is prior to laws of nature.

People use sentences to express propositions. People express propositions they state when making statements; they also express propositions they do not state. For example, the proposition that zero exists is expressed both in stating that zero exists and in asking whether zero exists.[4] A proposition can be entertained in many ways. What we investigate, doubt, assume, postulate, believe, and know are often propositions. One and the same sentence is routinely used on different occasions to express different propositions. Likewise, one and the same proposition is commonly expressed by different sentences in different languages and, quite often, even in the same language.[5]

Differences between sentences and propositions are involved in describing differences between direct and indirect quotation. In some cases, when we quote directly we intend to give the exact sentence quoted without conveying the proposition expressed. Abe said: "She loves him". In some cases, when we quote indirectly we intend to give the proposition expressed without conveying the sentence used. Abe said that Betty loves Carl. We can quote a person directly and without awkwardness add "But I did not understand what was meant". Quoting indirectly presumes understanding the proposition indirectly quoted; quoting directly does not.[6]

Although both propositions and sentences are abstractions, some sentences can be seen: when we read an *inscription* or *token* of a sentence we see the sentence—not the proposition, if any, the sentence is used to express. Sometimes we can look at an inscription for some time before seeing the sentence inscribed. Most sentences have not been inscribed and are thus not seen: such sentences will not be seen until inscribed and they will not be seen when all inscriptions are obliterated.

Ontology is important in this paper as shown in the following considerations. Socrates taught Plato. My judgment that Socrates taught Plato was made many years ago. My statement of the proposition that Socrates taught Plato was made seconds ago using the sentence 'Socrates taught Plato', which involves three English words but which contains no meanings and no persons. The proposition that Socrates taught Plato involves three meanings but it contains no words and no persons. The fact that Socrates taught Plato involves two persons but it contains no meanings and no words. In the senses recommended, facts, propositions, judgments, sentences, and

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[4]To the best of my knowledge, this type of observation is due to Frege (*1918/1956*, 294).

[5]The last few points have been standard for years. The result of adding 'in fact' and a comma to the front of a sentence expressing a given proposition is a different sentence expressing the same proposition. See Cohen-Nagel *1934/1962/1993*, xxii–xxv, 27–35.

[6]Indirect quotation is a subspecies of indirect discourse (Audi *1999*, 424).

statements comprise mutually exclusive ontological categories. However, questions of "ontological status" play no role in this paper: it is irrelevant whether sentences, propositions, or facts—or for that matter, characters, numbers, or truth-values, are "real entities"—are "idealizations", or are "theoretical constructs".[7] As is suggested by the title, this paper is written to be read aloud.

## 2   Sentences, Judgments, Statements

> A sentence is made up of words; a statement is made in words.—Austin *1961*, 88.
> Statements are made; ... sentences are used.—Austin *1961*, 88.

Since context is important in speech, it might help to begin with examples. Along the 3000-mile northern border of the United States with Canada, the weather is changeable, to say the least. The weather is a frequent topic of conversation. There is keen interest not only in the weather itself, but also in the nature of communication about weather, in weather discourse.

Perhaps interest in weather discourse is increased by the fact that Americans report weather in one system of units while Canadians report it in another. Americans report temperature in Fahrenheit degrees, where thirty-two is freezing. Canadians report temperature in Celsius degrees, where zero is freezing but thirty-two is very hot. People on one side of the border converse with friends and relatives on the other. It is not unusual for someone to get a weather report in one system and convey its content to others in another system. This situation gives rise to sentences that may seem strange in other contexts.

Here are some examples.[8]

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[7]I have discussed such questions elsewhere (e.g. *2006s*).

[8]As indicated in the abstract, this paper is primarily meant to be read aloud and then later to be discussed as a written text. As usual, the reader must "impersonate" the author. In order to make the text look "normal", instead of displaying a sentence as such, e.g. 'ten is fifty' (no uppercase first letter, no period), I display the so-called *assertoric* form of it which begins with an uppercase tee and ends with a period, 'Ten is fifty.'(sic) in this case. Where logic matters, this becomes important. The sentence 'ten is fifty' is literally a part of its own negation 'it is not the case that ten is fifty', but the assertoric form of the sentence is never a part of the negation of the sentence, or of the assertoric form of the negation of the sentence. Moreover, the sentence is part of its quotes-name and the assertoric form of the sentence is part of the quotes-name of the assertoric form of the sentence. But neither is part of the quotes-name of the other. Tarski (e.g. *1969/1993*, 103) is one of the few writers who usually keep the sentence apart from its assertoric form. And he never explains what he is doing or why. See Corcoran *2006s* for more details.

Zero is thirty-two.
Ten is fifty.
Twenty is more than sixty-five.
Twenty is between sixty-five and seventy.
Zero is to one hundred as thirty-two is to two-twelve.
Both hundreds are hot but only one is boiling.

Even without conversions between Celsius and Fahrenheit, weather discourse sometimes involves sentences that are otherwise unusual.

Zero is freezing.
Ten is cold.
Twenty is comfortable.
Thirty is hot.
Forty is sweltering.
Freezing is zero.
Freezing is thirty-two.
Thirty-two is freezing.
Thirty-two is hot.
One-hundred is boiling.
We can swim in eighteen.

In doing conversions from Celsius to Fahrenheit, it helps to know that ten Celsius degrees is eighteen Fahrenheit degrees. Zero, ten, twenty, and thirty degrees Celsius are respectively thirty-two, fifty, sixty-eight, and eighty-six degrees Fahrenheit. This is calculated by adding eighteen Fahrenheit degrees whenever ten Celsius degrees have been added.

Notice that ten degrees Celsius is a definite single point on the Celsius scale whereas ten Celsius degrees is not a point but an interval-size that has no location on the scale. It would be *incoherent*, a so-called *category mistake*, to say that today's temperature is ten Celsius degrees or that the difference between yesterday's high and low was ten degrees Celsius.[9] A person saying something incoherent is sometimes making a category mistake.

I have been using sentences made up of English words to make statements based on judgments. In the words of Austin quoted above: "A sentence is made up of words; a statement is made in words". In several places Frege

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[9]Of course, if someone writes the expression 'the current temperature is ten Celsius degrees' we should not jump to the conclusion that there was a category mistake and not just an inadvertent writing error, a transposition of words. We could ask whether they intended to write 'The current temperature is ten degrees Celsius'. An affirmative answer would suggest a writing mistake and not a category mistake. Notice that the quotes name of the assertoric form of a sentence does not have a period at the end unless it occurs at the end of the sentence using it.

said that the answer to a question is a statement based on a judgment (*1918*, *1919*, *1952*, 117; *1997*, 299).

The statements, or assertions[10], that I have just made aloud in this spoken delivery were based on judgments that I made earlier, silently of course. Some judgments were made as I was planning the paper; some were made long before that. Some judgments involved little beyond confirmation of memory; some involved perception; some calculation; some deduction; some induction. Every objective judgment is made in reference to the fact it concerns (Corcoran *2006e*). The statements were all public and were all made in public today. The judgments were all private and were all made in private before today. No two persons can make the same judgment.

In judging I form a fresh belief, often a completely new belief in the truth of a proposition not previously believed by me—perhaps even previously disbelieved by me. In stating per se new beliefs are not formed: some statements are not based on belief at all; some are contrary to their speaker's judgments—a fact repeatedly overlooked by Frege.

In the senses recommended, 'judgment' and 'statement' are common nouns whose extensions are classes of events: epistemic events in the case of 'judgment'; pragmatic events in the case of 'statement' (cf. Austin *1961*, 87). But the ending 'ment' brings other senses. The two words can be used as proper names in various senses. For example, 'judgment' can be used to name the human faculty by which judgments are made or with qualification it can be used to name a person's faculty as 'Abe's judgment improved in time'. Both can be used as common nouns for the objects or results of the eponymous acts in various senses: Abe's judgment (statement) [sc. the proposition Abe judged (stated)] was the same as Ben's.

## 3   People Use Words to Mention Things.

> When saying something of an object, one always uses a name of this object and not the object itself, even when dealing with linguistic objects.—Alfred Tarski *1969/1993*, 104.

> Premises rhymes with nemesis, not with nemeses.—Frango Nabrasa (per. comm.)

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[10]I use the word 'assertion' reluctantly as a synonym for 'statement' in the recommended sense. However, the fact that it has drastically different senses even in logic calls for caution. Some logicians use it to referred theorems, statements having the highest level of warranted assertibility. But it is widely used outside of logic for statements with the lowest level of warrant. At the end of a famous address before the United Nations, Colin Powell asserted that his previous statements were not assertions. Other uses occur, e. g. Quine *1970/1986* and Goldfarb *2003*. See Quine's insightful quip (ibid. p. 2).

After this section of the paper, I may seem to be at a slight disadvantage. Up to this point I have been using some spoken sentences to mention other spoken sentences. The sentence I am now reading is a spoken sentence.

After this paragraph I will of course continue to *use* only spoken sentences, but I will be *mentioning* mainly written sentences. From here on, most of the sentences that will be mentioned are written sentences.[11] The sentence I am now reading is a written sentence.

I will not be able to use what I will be mentioning. Isn't that the usual case? No one uses the freezing point to mention the freezing point.

However, we routinely use expressions that we are mentioning. As I noted above, I used spoken sentences to mention spoken sentences. Before the invention of writing there was no other way to mention spoken sentences.

As we saw above, it is common to encounter an ambiguous sentence, one that could normally be used to express two or more propositions. When I read an ambiguous sentence to an audience, one listener might have a *thought* of one proposition at the precise time that another listener has a thought of another proposition. When I said 'The sentence I am now reading is a spoken sentence', some readers thought that the proposition thereby stated was false and some thought that the proposition stated was true. The expression 'the sentence I am now reading' is ambiguous. Some readers might have thought that I was talking about a sentence written on the paper I am reading from; others might have thought that I was talking about the sentence I was enunciating.

Using one ambiguous expression twice in consecutive paragraphs intending different meaning each time is a practice to be avoided, other things being equal.

Of course, although no two persons can have the same *thought*, thankfully nothing prevents their separate thoughts from being thoughts of the same proposition. Otherwise, all agreements and disagreements would be merely "verbal" and communication would be of a very restricted nature, if indeed it could go on at all.

Occasionally, two of the propositions expressed by one ambiguous sentence are of such a nature that in expressing one we use a word that we mention when expressing the other. For example, consider the three-word sentence 'Santiago is Spanish'. In one sense, this expresses a proposition to the effect that the city of Santiago de Compostela is in Spain. The eight-

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[11]Thus, every sentence is a string of characters, counting the space or blank as a character. But, of course, not every string of characters is a sentence. The terminology varies, even within the writings of one author. Tarski says that a sentence is a series of printed signs (*1956/1983*, 156). Church writes of a sentence as being a succession of letters (*1956*, 27). Boolos writes of sequences of symbols and of strings of symbols on the same page (*2002*, 103).

letter word 'Santiago' is used (to mention the city), but the word itself is not mentioned. In another sense, the same three-word sentence expresses a proposition to the effect that the one-word expression 'Santiago' is part of the Spanish language; it translates as 'Saint James'. Here the word 'Santiago' is mentioned (by using the same word, 'Santiago' itself). In a third sense, it expresses the proposition that Saint James is Spanish, which is false, of course. Here again the word 'Santiago' is used (but to mention the saint). Before leaving this important topic let us see what Church says (*1956i*, 61).

> Following the convenient and natural phraseology of Quine, we may distinguish between *use* and *mention* of a word or symbol. In 'Man is a rational animal', the word 'man' is used but not mentioned. In 'The English translation of the French word *homme* has three letters', the word 'man' is mentioned but not used. In 'Man is a monosyllable', the word 'man' is both mentioned and used, though in an anomalous manner, namely autonymously.[12]

Notice that the last spoken word quoted is ambiguous; the sense intended by Church was coined by Rudolf Carnap in 1934 (*1934/1937*, 17). The third syllable of its written counterpart is spelled en-wye-em as is the case in the rhyming word 'synonymously'. It should not be confused with the phonetically similar and older 'autonomously' whose third syllable is spelled en-oh-em (Chateaubriand *2005*, 150). The two contextually homophonic third syllables *nym* and *nom* are not etymologically related - any more than the number-word 'two' is related to the preposition 'to' with which it is a homophone.[13]

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[12]Church (*1956*, 61). The italicization is Church's, but Church uses double quotation marks where single quotation marks are used above in keeping with the style of this article. In the same book, actually on the next page (*1956*, 62), Church uses single quotation marks in mentioning words and he even explicitly notices the advantage of using single quotations in this way.

[13]As far as I know, no meaning has yet been assigned to 'synonomy', 'synonomous', or 'synonomously'.

# 4   Written English is not a phonetic transcription of Spoken English.

> Now spoken sounds are symbols of mental ideas, and written marks symbols of spoken sounds. Just as written marks are not the same for all humans, neither are spoken sounds. But what the latter are symbols of - mental ideas - are the same for all; and what these ideas are likenesses of - actual things - are also the same—Aristotle, *On Interpretation.*[14]

The adjectives 'spoken' and 'written' are ambiguous: each has a broad and a narrow sense—like the word animal. In the broad sense, every human is an animal; in the narrow sense, no human is an animal. There are many spoken sentences (broad sense) that may never have been spoken or uttered by anyone and which therefore are not spoken sentences (narrow sense). Likewise, many written sentences (broad sense) have never been written or inscribed by anyone and which therefore are not written sentences (narrow sense).

For example, the two-word sentence 'zero sleeps' made up of the four-letter word 'zero', or zee-ee-ar-oh, and the six-letter word 'sleeps', or es-el-ee-ee-pee-es, may never have been written or inscribed by anyone before, but even before it was written it was a written sentence.[15] Of course, any proper name can be written or inscribed one space in front of an inscription of an intransitive verb in order to make an inscription of a written sentence. You can picture the following examples of written sentences.

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Arithmetic boils.

Geometry freezes.

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[14]This translation of *On Interpretation*, I, 16a3–8 is essentially the one used by N. Kretzmann (*1974*, 3–4) in his informative and accessible discussion of this passage, which he calls "the most influential text in the history of semantics". Kretzmann credits the translation largely to Ackrill *1963*.

[15]This is the first spoken sentence today illustrating two of the ways of making spoken proper names of words: the appositional method is to say the two-word expression 'the word' just before pronouncing the word itself, the phonetic-orthographic method is to "spell-out" the word, i.e. to pronounce the names of the word's letters in the order in which they occur in the word. Thus, the word 'zero' is one and the same thing as zee-ee-ar-oh. These two spoken alternatives correspond somewhat to the two written ones in Tarski's truth-definition paper: the quotation-mark name and the structural-descriptive name (*1956*, 156).

Have inscriptions of these sentences ever existed before? Who could know except someone who saw such an inscription?

A written sentence is not necessarily a sentence that has been written but rather a sentence of the kind that *could* be written. In other words, a written sentence is a string of characters.[16] In our case, the characters are all either letters of the Latin alphabet or punctuation marks. Is the space that occurs between consecutive words or between consecutive sentences a character? Or is it a non-character? Or is it a borderline case?

It is certainly significant; it is used meaningfully even though it is not used to express a constituent of a proposition, a concept. The word 'character' is in many ways typical of the words we will be focusing on: it is ambiguous and some of its meanings are vague. In the sense recommended in this article, the space is a character. Ambiguity (or polysemy) and vagueness (or indefiniteness) are two of the most pervasive of linguistic phenomena. My advice is to recognize them and to become accustomed to them.[17] A writer might use an ambiguous expression without being equivocal. A writer might use a vague concept without being guilty of imprecision. Aristotle tells us not to strive to be more precise than the subject-matter permits (*Nicomachean Ethics*, I, 3). Describing a sunrise requires vague concepts.

Written sentences are things I will be mentioning but not using in this spoken presentation. The sentences that I will be using are strings of sounds, in one sense of this ambiguous word. Using the perspicacious terminology coined by Charles Sanders Peirce, a written sentence is one that can only be *embodied* visually and a spoken sentence is one that can only be *embodied* audibly. The linguists who know about such matters conclude that spoken English and written English are such disparate systems of communication

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[16]The problem of finding an exact definition of "Written English sentence" is far from settled; it is one of the famous unsolved problems in contemporary linguistics. Noam Chomsky brought this and related problems to the forefront of linguistic research. Likewise, even the problem of finding an exact definition of "Written English word" is far from settled (Lyons *1977*, Vol. I, Ch.1). The word 'word' is of course ambiguous but, in the "typographical" sense used in this article, a word is nothing more than a string of characters. Thus, there are no such things as homonyms, or more precisely, homo*graphs*. No *two* words have the same spelling, or the same succession of characters. A word is its spelling, so to speak; "two" words spelled the same are one. And no *two* sentences have the same wording, or the same succession of words and spaces.

[17]The "logically perfect" or "formalized" languages studied in mathematical logic exhibit neither ambiguity nor vagueness—this is an advantage for some purposes, and a disadvantage for other purposes. (Cf. Church *1956*, 2, 3, 32, 50.) There are of course needless ambiguities, sometimes called misnomers, which are exasperating because they entered the language by way of a mistake involving what had been a misuse of a "less ambiguous" word. The stock example is the word 'Indian' whose misuse became so widespread that it had to be recognized as a use. There are many other examples, several in philosophy and logic. Church cites 'sentence' as a possible example (*1956p*, 6).

that the two should not be regarded as different forms of the same language but rather as two different languages. My language, [American] English, is actually two languages.[18]

In keeping with the usual convention of capitalizing proper names of languages, I will be using Spoken English (written as a proper name) to discuss Written English (also with capitals); henceforth, unless explicitly indicated otherwise, the single word 'sentence' is elliptical for the two-word expression 'Written sentence'—in the broad sense of a writable sentence, not one that has necessarily been written.[19]

There is no such thing as an English sentence that has two "forms"—a "Spoken form" and a "Written form". Every English sentence is a sentence of Spoken English or of a Written English, and not both, of course.

Using the terminology of modern logic, we can say that in the discussion to follow, Spoken English is the *metalanguage* and Written English is the *object-language*. Actually I should say '*an* object-language' or 'one of the object-languages', not '*the* object-language', because I will be mentioning Spoken English sentences also, as well as sentences of a few foreign written languages. In our case, Spoken sentences of *the* metalanguage will be used by a speaker, namely me, to mention mainly Written sentences of one object-language, Spoken sentences of another object-language, and written sentences of other object-languages. More importantly, I will be using sentences to make statements about object-language sentences to the members of the audience, namely you, who are responsible for critical evaluation. The members of the audience routinely make judgments[20] about the speaker's

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[18]It is obvious that Written English is not a phonetic representation of Spoken English. There are no Spoken sight rhymes and no Written sound rhymes. Moreover, Written English can represent logical groupings with parentheses and other typographical devices where Spoken English is without equally adequate devices. On the other hand, there are propositions expressible in a small number of words of Spoken English that can not be expressed in Written English at all. In these cases, every Written sentence expressing a proposition implying one intended by such a Spoken sentence also implies propositions not implied by the intended one. The gulf between the two languages becomes even more evident if we try to find a pair of sentences one Written and one Spoken which are uniformly translations of each other in the sense that they express exactly the same range of messages. To the question of whether there is a peculiarly English system of messages that can only be exactly conveyed in one or both of the two media Spoken English and Written English, I would answer yes, tentatively and with qualifications.

[19]Accordingly, the two two-word expressions 'Spoken sentence' and 'Written sentence' are used as ellipses respectively for the two three-word expressions 'Spoken English sentence' and 'Written English sentence'.

[20]Gottlob Frege, the meticulous logician regarded as the founder of Analytic Philosophy, uses the word 'judgment' in a sense close to that used here. He says (*1980*, 20): "According to my way of speaking, we think by grasping a thought, we judge by recognizing a thought as true, and we assert by making a judgment known [to others]". Also see Frege *1918/1956*, 294. However, not everyone is this careful. For example, David

statements and about the subject matter of the speaker's statements. My next statement is a good example to reflect on.

> It is obvious that the Written English sentence 'zero is cold' can be used to express a true proposition if and only if zero—in some sense—is cold—in some sense.

By using this Spoken sentence, I mention but do not use the Written sentence 'zero is cold'; and I use but do not mention the Spoken sentence *zero is cold*.

There is no way to confuse a statement with a sentence or with a proposition, since it is only the statement that inherently has a speaker, who is responsible for its accuracy, and it is only the statement that per se has an audience, which is responsible for its critical evaluation. Every statement is an event. Like every other event, every statement has effects, ramifications which are also events, not propositions; and like any other action, it changes the world in which it is made. No proposition is an event. Every proposition has consequences, implications which are propositions, not events; the proposition per se is causally inert. No statement is a proposition. No proposition is a statement. Here I am using the standard, traditional terminology that Church so adequately describes in his classic 1956 article "Propositions and Sentences", perhaps the clearest thing ever written on the topic, certainly the best thing I know of (*1956p*, 3–11). To be explicit, the word 'statement' is used in the *transactional* sense, the word 'proposition' in the *intensional* sense, and the word 'sentence' in the *syntactic* sense.

## 5    Sentences Express Propositions and People Make Judgments.

> When a sentence expressing a proposition is asserted we shall say that the proposition itself is thereby asserted.—Church, *1956i*, 27.

> It seems to me that [by imagination vs. judgment] you have in view the difference between grasping a thought [sc. proposition] and recognizing a thought as true. The latter is what I call judging.—Frege to Russell in 1904. *1980*, 163.

Any one sentence—made of words and ultimately of letters and characters—can be used to express two or more different propositions or, more generally,

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Hilbert uses the word 'judgment' to mean "proposition", in a sense close to that of this paper (Luce *1950*, 165–6, Hilbert-Ackermann *1928/38/50*, 171).

different *messages*—made up of meanings or concepts. The number of words in a sentence is not usually the same as the number of concepts in a message it expresses; it could be more but it is often less. Aristotle had already said that although sentences are not the same for all humans, what they express is the same for all ( *On Interpretation*, Ch. 1, 16a3-18).

The three-word sentence 'thirty-two is freezing' is typical. It can be used to express the true proposition that thirty-two degrees Fahrenheit is the freezing point of water. Thirty-two *is* freezing. I just made a statement that thirty-two degrees Fahrenheit is the freezing point of water by using the three-word sentence. The very same sentence can also be used to express the false proposition that thirty-two degrees Celsius is the freezing point of water. I never made a statement to that effect. It can be used to express a proposition about one of the athletes known as thirty-two.

And it can be used to express an incoherent *message* to the effect that the number thirty-two is in the process of becoming frozen, or perhaps that the number thirty-two feels cold. I am using the word 'message' for the genus of what is expressed by a complete sentence. Thus a message is either a proposition, whether true or false, or an incoherency, which is neither. An incoherent message cannot be coherently said to be a mistake any more than a false proposition can coherently be said to be a lie. Some incoherencies can be poetic.[21]

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Uncomfortable Numbers: A Poem.

Thirty-two is freezing.
Thirty-two is hot.
Thirty-two is sweating.
Thirty-two is not.

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The above paragraph including the poem illustrates several facts. First, sentences are ambiguous, or polysemous: rarely if ever is a sentence unambiguous, or monosemous. Second, propositions are about something and it

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[21]There are several fallacies that arise in discussions of incoherencies, i.e. incoherent messages. Perhaps the most common is to think that there are sentences that somehow by nature can only be used to express incoherencies. The fact is that people can make conventions instituting new coherent uses for any given sentence. Power corrupts. Glory addicts. Fame fades. Logic confuses. Logic clarifies. Another common fallacy is to think that a sentence expressing an incoherency is meaningless. The expression 'meaningless sentence expressing' itself understood in the intended way is an oxymoron: every sentence expressing something means that which it expresses. Another common fallacy is to confuse contradictory messages with incoherent messages. The message that some odd number is not odd is coherent but contradictory and thus false. The message that some odd number trisects a right angle is incoherent, thus not false, and thus not contradictory.

is in reference to their subject-matter that they are true, or false, as the case may be.[22] Third, not every meaningful use of a sentence involves using it to express a proposition; a sentence can be used to express an incoherency, a message that is neither true nor false.[23] Fourth, not every pair of sentences that could be used to express a contradiction are so used in a given discourse—there are "verbal contradictions" that are not contradictory. We have already seen that it is not necessarily self-contradictory or even false to say that not all written sentences are written sentences, nor is it necessarily tautological or even true to say that all written sentences are written sentences. Logicians and grammarians might legislate that no one expression may be used in two or more senses in one text. Even though their advice is generally sound, their jurisdiction hardly ever extends beyond the classroom.

The established notational convention already applied above is to use single quotes for making names of sentences and other expressions, for example, names of characters, words, phrases, and sentences. These are the so-called single-quotes-names.[24] Thus 'one plus two is three' is a five-word English sentence and 'square' is a six-letter English word, both recognized by me, neither of which would have been recognized by Aristotle. Following Bertrand Russell, Rudolf Carnap, Morris Cohen, Ernest Nagel, John Lyons and others, double quotes are used in naming propositions, incoherent messages, and other meanings.[25] These are the so-called double-quotes-names. Thus, "one plus two is three" is a true proposition known both to me and to Aristotle and "square" is a concept also well known to both.

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[22]According to this traditional conception of proposition it does not seem coherent to say that a proposition is true *in*, *at*, *on* or *of* a possible world, a time, a circumstance, a situation or anything else; a proposition is either true or false. This is of course not to say that the word could not be used in some other sense according to which it is coherent to say, e. g. that some propositions true in this world are false in some other world, although in this case it would be hard to imagine how such an apparently inaccessible fact could be known. Frege, Moore, Church, and Austin are silent on these senses of 'proposition'.

[23]The question arises whether incoherent messages have the same objective existence that belongs to propositions or whether they are entirely subjective, merely mental constructs. This is a difficult issue which may turn out to be as much a matter of convention as of fact.

[24]Church (*1956p*, 61) says that Frege used single quotes to indicate that the quoted material was being used autonymously; this is *not* using single-quotes-names. In diametrical opposition, both Tarski and Quine regard a sentence using an occurrence of a quotes name as not using the quoted expression at all. Consider the next sentence in the body of the paper; it mentions 'square'. Frege regarded it as using the 6-letter word 'square'; Tarski and Quine regarded it as using the 8-character quotes-name of the 6-letter word but not using the 6-letter word at all.

[25]*Bertrand Russell 1903*, 53ff, *1905/1967*, 99; *Carnap 1934/1937*, 14; *Cohen-Nagel 1934/1962/1993*, xxii; *Lyons 1977*, Vol. I, x.

In simple cases, expressions [are used to] express meanings or senses and they [are used to] name, or mention, entities or things. Thus the number-word 'three' expresses the individual concept "three" and it names the number three, the number named 'three'. In some cases the situation is more complicated: the sentence 'one plus two is three' expresses the proposition "one plus two is three"; but it is incoherent, even ungrammatical, to say that it names one plus two is three. The expression 'three' expresses the concept "three". This is a pattern often safe to follow. The expression 'three' names the entity three. This pattern is less safe; it must be used with care.[26]

The expression 'no number' denotes no number, but 'some number' does not denote some number. In fact, no number is named 'some number', or for that matter 'no number'. Of course, in a heteronymous sense, the expression 'no number' does not denote at all, a fortiori it does not denote something named 'no number'. There is nothing named 'no number' except the expression itself, which is no number. In the autonymous sense of 'no number', no number denotes no number, which is an expression not a number. Likewise in the autonymous sense of 'no number', no number denotes nothing, thus not an expression, a number, or anything else.[27]

There are several other terminological systems that are in use; some may be more appropriate in certain contexts. All must be used with care. Some writers say that an expression expresses, connotes, or signifies an intension, a connotation, or a meaning, or a sense. And, in contrast, they say that an expression names, refers to, or denotes an extension, a referent, or a denotation. Some writers take "the connotation" of an expression to be its emotional overtones and explain that it may vary from person to person even when there is agreement on what is expressed and on what is named. The expression 'seven' taken by two people to express "seven" and name seven may connote good luck for one person and bad luck for another.

There is little chance to confuse a judgment with a sentence or with a proposition. Each judgment is an event; it is an act done at a particular time by a particular person. The person judging that a certain proposition is true usually grasps the proposition before judging that it is true. I remem-

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[26]Church (*1956p*, 25f.) follows Frege in holding that, in a logically perfect or formalized language totally devoid of ambiguity, all sentences expressing true propositions name an entity *truth* and all expressing false propositions name an entity *falsehood*. This "discovery" by Frege was unprecedented in the entire history of logic and linguistics and, as far as I can tell, it has yet to be widely accepted. My guess is that in the fullness of time it will be relegated to a footnote in the history of ideas.

[27]Contrary to the impression given by Russell in his 1905 paper "On Denoting", most of the expressions he calls denoting phrases do not denote in any sense of 'denote' I know of—certainly not in the sense of Church *1956p* used in this paper—except when used autonymously.

ber clearly that before concluding that thirty degrees Celsius is eighty-six Fahrenheit, I considered the proposition and wondered whether it was true. Before doing the mental process of verifying by calculation the premise that thirty-two plus three times eighteen is eighty-six, I considered the conclusion as a hypothesis, consciously suspending judgment. You might have done the same. As mentioned above, neither the proposition nor the sentence used to express it has a time of occurrence.[28] These facts are independent of the fact that the word 'judgment' exhibits what is known as process/product ambiguity—besides being used for the action or process of concluding that a proposition is true it is also used for the result or product, for the proposition concluded, the conclusion of a judging. The same ambiguity belongs to the word 'conclusion'.

There is little chance to confuse a judgment or conclusion with a statement or assertion although they are both actions. Both require a proposition but the judgment does not require a sentence. Perhaps you made a judgment or arrived at a conclusion and then later decided which sentence or language to use to express it. Both require an agent but no judgment can have an audience.

When the words are not used in the senses recommended there is much room for confusion. The same process/product ambiguity belongs to the word 'statement'—besides being used for the action or process of stating that a proposition is true it is also used for the result or product, for the proposition stated, or even the sentence used. When the words are both used in the product sense, we have the sorry spectacle of saying correctly that the judgment is a proposition, the statement is a proposition, and the judgment is a statement. Even though there is nothing objectively wrong about using ambiguous words, or even using them in multiple senses in one and the same paragraph, nevertheless many able writers have confused their readers by doing so.

In the above product senses of 'judgment' and 'statement' there is nothing incoherent about saying that a judgment or statement is true or is false. There are other such senses as well: the sentence 'The statement you made is true' can be taken as elliptical for 'The [proposition asserted in the] statement you made is true'.

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[28] Absurdly, the word 'judgment' (French 'jugement') has been used in logic since at least the early 1600s for the fictitious act of simultaneously constructing a proposition, determining that it is true, and asserting it (*Arnauld and Nicole 1662*, Part II, Ch. 2 and esp. Ch. 3,: *Whately 1826*, 57, 75–77). Whately can not conceive of a non-judged proposition; he thinks that the copula 'is' or 'is not' "indicates the act of judgment" (*1826*, 57). These writers overlook the fact that every day we consider propositions not known to be true and not known to be false. The non-judged hypothesis that the defendant is the murderer is often grasped by the jury even before any evidence can be presented.

# 6   Sentences, Not Propositions, Are Ambiguous or Unambiguous, Extrinsically; Propositions, Not Sentences, Are True or Untrue, Intrinsically.

> A proposition is composed not of words, nor yet of thoughts, but of concepts.—G. E. Moore *1899*, 179.

The words 'sentence' and 'proposition' are both ambiguous. Besides the syntactic, typographical, or morphological meaning explained above, the word 'sentence' can also be used as elliptical for 'meaningful sentence' in the hybrid, or composite, sense of a sentence expressing a certain one of the messages it normally expresses. This would be a sentence meant or taken in a certain way; in some cases, one might say a *propositional sentence*. Likewise, besides the intensional sense explained above, the word 'proposition' can also be used in a hybrid sense to indicate a message or a proposition expressed in words in a certain way. The latter would be a proposition expressed a certain way, a *sentential proposition*. There is little more than emphasis to separate the hybrid sense of 'sentence' from the hybrid sense of 'proposition'. This is brought out by the fact that 'sentence' in its hybrid sense was used by Mahoney to translate the German word 'Satz' as used by Frege (*1884/1964*), while 'proposition' in its hybrid sense was used by Austin to translate the same passages (*1884/1964*). Frege himself recognized an ambiguity as did Hans Kaal, the translator of the Frege correspondence, who agrees with Austin. Frege's letter to Russell of 20 October 1902 (*1980*, 149) contains the following passage:

> Your example ... prompts me to ask the question: What is a proposition? German logicians understand by it the expression of a thought [sc. proposition], a group of audible or visible signs expressing a thought [sc. proposition].[29] But you evidently mean the thought [sc. proposition] itself. This is how mathematicians tend to use the word. I prefer to follow the logicians in their usage.

In fact, Tarski often used 'sentence' in the sense "meaningful sentence", i.e., for propositional sentence. Church discussed the use of 'proposition' in the sense "sentential proposition". In this sense, a proposition is "a composite entity, sentence plus proposition" (Church *1956p*, 6). But he

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[29]I am reluctantly following the established practice of reading 'proposition' where a strict and literal translation of Frege would require the grossly inappropriate word 'thought'. However, it is clear to me that Frege's "thoughts" are not my propositions. Two sentences expressing the same thought need not express the same proposition. Consider the following scenario. Abe called yesterday. Also yesterday, I used the sentence 'Abe called today' to report the fact that Abe called yesterday. The proposition expressed using 'today' is not the same as the one expressed using 'yesterday' but the thoughts are the same (*1918*, 296).

used the word only in the abstract intensional sense of this paper, i.e. for a combination of meanings that is either true or false. Boolos (*2002*, 112, 114) used the word 'sentence' as a technical term only in the purely syntactic, typographical sense stipulated here in which a sentence per se is a "syntactic entity", a string of characters without regard to whether it has been used to express a message or to which meaning has been or will be assigned to it by writers.[30] He also used it casually in the hybrid sense (*2002*, 103).

In order to eliminate one source of confusion, slashes (virgules) can be used to refer to composite entities. The composite word /one/ is a two-part system composed of the string 'one' and the sense "one". The number one is denoted by the word /one/ and determined by the sense "one". The composite /one plus two is three/ is "a composite entity, sentence plus (abstract) proposition" (Church *1956p*, 6), that expresses or *contains* the proposition that one plus two is three. As Chateaubriand (*2001*, 379) points out, composites such as /one/ give rise to what has been called a *semiotic triangle*: the three vertices are respectively a string, a sense, and a referent and the three sides are respectively the relations of string-to-sense, string-to-referent, and sense-to-referent. He says that a composite expression "contains" its sense.

I used the word /Santiago/ autonymously above in this article. As already implied, the word /autonymous/ coined by Rudolf Carnap in 1934 should not be confused with the phonetically similar and older /autonomous/ which is not etymologically related (Chateaubriand *2005*, 150).The slash notation cannot be used unless it is clear in the context which of the various interpretations is intended as in:

The number one is denoted by the composite word /one/.

A string per se is meaningless: nothing is denoted by the string 'one' except under an interpretation by a person. In many ways it would be graceful to use something more familiar such as italics to mention composites.[31]

The composite word *one* composed of 'one' and "one" denotes the number one.

In both of the two senses mentioned the word 'proposition' is a technical term of logic and has been for centuries. Church wrote (*1956p*, 3):

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[30]The logician Warren Goldfarb used the word 'statement' for something very close to an idealized sentence-proposition (Goldfarb *2003*, 5ff.). Quine (*1970/1986*, 13–14) used the expression 'eternal sentence' for this. See the comments below about Quine's most mature terminology.

[31]The disadvantages include the ambiguity resulting from the fact that italic font has other uses and the fact that italic can not be italicized: //one// is /*one*/ or /*one*/ and is composed of '/one/' and "/one/".

In Latin, *propositio* was originally a translation of the Greek *protasis*, and seems to have been used at first in the sense of premiss.

In either technical sense it has no meaningful connection with a proposing or a proposal—just as the sense of the word 'work' as a technical term of physics has no meaningful connection with employment or labor. Some logicians avoid using the word 'proposition' as a technical term but nevertheless say many things that I would say using that word. Some simply substitute a different word, and not always the same different word. Tarski sometimes used 'thought' (*1956/1983*, 160), sometimes 'judgment' (*1969/1993*, 106), and sometimes 'fact' (ibid. 104). Sometimes Tarski simply avoided the concept "proposition" by studiously using the word 'sentence' in his favored sense of "propositional sentence"(*1986*, 143–154, *1996*, 119–130).

According to Tarski: the sentence 'zero is even' is true if and only if zero is even. But that would be incoherent in the terminology of this paper. According to this paper: the proposition "zero is even" is true if and only if zero is even. Likewise, the sentence 'zero is even' is used to express a true proposition if and only if it is used in a sense in which zero is even.

In the purely intensional sense of 'proposition', it is incoherent to speak of the meaning or of the wording of a proposition—but in the hybrid sense both make perfect sense. Likewise for 'sentence', in the pure syntactic sense, it is incoherent to speak of the meaning or of the wording of a sentence—whereas in the hybrid sense both make perfect sense. Accordingly, in the hybrid senses it is incoherent to speak of ambiguous propositions or of ambiguous sentences.

As indicated above, in the purely syntactical sense advocated in this work, a sentence is ambiguous if there are two or more meanings that it normally used to express. And there is no such thing as a *pair* of sentences composed of exactly the same characters in exactly the same order: a sentence *is* a series of characters. However, in the hybrid sense, often but not always used by Tarski, a sentence is ambiguous if there is *another* sentence composed of exactly the same characters in exactly the same order but having a different meaning. And there is no such thing as one sentence having two meanings: a sentence *is* a meaningful unity. A hybrid sentence is a compound unity uniting two parts: it is a pure sentence plus one of its meanings. Many modern logicians seem to use the word 'sentence' usually in the pure sense as is suggested in the common locution that a sentence is only true or false under an interpretation. Many philosophers follow Tarski in usually using the word in the composite sense as suggested in the common locution that not every true sentence is known to be true. But, there are other important senses to be recognized.[32]

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[32]Frege (*1918*, 292–4, passim) uses it in a sense very close to "spoken statement". He

The notions of sentence and proposition are both useful for discussing various phenomena including translation and ellipsis. In order for a sentence used to express a certain proposition to be elliptical it is sufficient for the sentence to omit a word corresponding to a constituent of the proposition. The following is elliptical under two of its normal interpretations: Russell read Frege more than Peano. Under one interpretation, it expresses the proposition that Russell read Frege more than Peano *did* [sc. read Frege]. Under another interpretation, it expresses the proposition that Russell read Frege more than *he did* [sc. Russell read] Peano.

## 7   Tokens, Not Occurrences, Embody Types. Occurrences, Not Tokens, Are in Types.

> A string-type has instances which are string-tokens or string-inscriptions composed of instances (not occurrences) of characters; the string-tokens are ultimately composed of character-tokens.—John Corcoran et al. *1974*, "String Theory".

Let us try a little thought experiment. Imagine that we have one copy of a Greek edition of Euclid's *Elements* on the left end of the table open to Proposition 47 of Book I, now known as the Pythagorean Theorem. In a line spreading to the right, imagine one copy each of ten perfect translations into ten different languages, say Arabic, Chinese, English, French, German, Portuguese, Spanish and three other languages that you may choose. Of course we imagine them all open to Proposition 47 of Book I. The English translation by Thomas Heath has the following: *In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.*

We have one proposition which is expressed by the Greek sentence written by Euclid. And we have eleven sentences each in a different language, all of which express one and the same proposition, namely the Pythagorean Theorem. The Pythagorean Theorem is a proposition in the preferred intensional sense used above. It is made up of concepts not words, not letters, not characters. This one proposition, called 'the Pythagorean Theorem', is that in virtue of which the ten translations of Euclid's Greek sentence are all perfect translations not only of Euclid's Greek sentence but also of each other. All eleven sentences express one and the same thing—and that thing is the proposition. In the hybrid sense, there are eleven propositions. A

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says that an indicative sentence is a series of sounds (ibid. 292) that contains not only a proposition but also the assertion of the proposition (ibid. 294).

person who does not recognize propositions in the intensional sense should not say '*the* Pythagorean Theorem' but rather '*a* Pythagorean Theorem' or '*one of the* Pythagorean Theorems (plural)'even when it is clear from the context that English is the language being used. After all there are many different ways of expressing a given proposition in a given language.

If persons who knew none of the eleven languages were to carefully examine all eleven sentences expressing the Pythagorean Theorem, they would find nothing to indicate a common meaning. Not even one character is found in all eleven. However, were someone to look at two copies of one and the same translation, say the English, turned to the same page, it would be easy to find an exact duplicate of the sentence token located in my copy of the Heath translation embodying a sentence expressing the Pythagorean Theorem.

In one sense of the word 'sentence' we have two English sentences, one in each of the two copies of the book—two concrete, visible things made of scattered bits of dried ink on paper. In another sense there is only one sentence, but it has been printed twice—one abstract ideal thing capable of being printed but not itself visible. The one thing printed twice (and capable of being printed indefinitely often) is what Peirce called a *type*, specifically a *sentence-type*. The two printings he called *tokens*, specifically *sentence-tokens*. The two sentence-tokens, in Peirce's terminology, are *embodiments* of the sentence-type. The two sentence-tokens came into existence at a certain definite time—maybe we could find out from the publisher the exact day. Sometimes the year of the printing is in the front of the book. But, when did the sentence-type, the abstract thing that there is only one of, come into existence?

Again, the two sentence-tokens can be destroyed by a fire in one library, or maybe by two fires in two libraries. But how could a sentence-type be destroyed? If someone destroyed all of the books, would that destroy the sentence-type? These questions are raised only to clarify the differences between types and tokens. Before exploring the type-token ambiguity[33] further, let us read Peirce's exact words. The following is from his 1906 *Monist* article (*1906*, 504–5) quoted in Ogden-Richards *1923*, 280–281.

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[33]Many nouns and noun phrases are subject to type-token ambiguity in that they can be used to refer to types or to tokens. See the previous paragraph. The type-token distinction is used to explain type-token ambiguity. Incidentally, a language that makes the type-token distinction need not have expressions that have the type-token ambiguity, and conversely. A sufficiently rich logically perfect metalanguage could have symbols for types and ways of mentioning tokens of those types but, of course, have no ambiguous expressions. A sufficiently primitive language could suffer from type-token ambiguity while lacking resources to discuss the distinction.

> A common mode of estimating the amount of matter in a ... printed book is to
> count the number of words. There will ordinarily be about twenty 'thes' on a page,
> and, of course, they count as twenty words. In another sense of the word 'word',
> however, there is but one word 'the' in the English language; and it is impossible
> that this word should lie visibly on a page, or be heard in any voice ... . Such a ...
> Form, I propose to term a *Type*. A Single ... Object ... such as this or that word
> on a single line of a single page of a single copy of a book, I will venture to call a
> *Token*. ... In order that a Type may be used, it has to be embodied in a Token
> which shall be a sign of the Type, and thereby of the object the Type signifies.

In my copy of Euclid's *Elements*, the concrete visible token of the sentence-type expressing the Pythagorean Theorem contains six concrete tokens of the word-type 'the', tee-aitch-ee. The sentence-token is spread out over three lines of print with two *thes* on the first line, three on the second, and one on the third. This means that the abstract sentence-type contains six *thes*, to use Peirce's clever word. Does this mean that a sentence-type is made up of sentence-tokens? Does this mean that something abstract, a sentence-type, is made up of concrete embodiments, sentence-tokens? We need another distinction here to deal with those questions and to make the point that, although the type 'the' has only one *occurrence* of the type 'e', the type 'e' *occurs* twice in the type 'these' and the type 'e' is instantiated, betokened, or embodied (to use Peirce's term) twice in every token of the type 'these', tee-aitch-ee-ess-ee. One and the same word-type 'the' occurs six times in the one sentence type expressing the Pythagorean Theorem. Every sentence-type is made up of character-types. Typically a sentence-type has multiple *occurrences* of at least one of it character types. Every token of a sentence type has the same number of tokens of a given word-type as the sentence-type itself has occurrences of the given word-type.

Some authors explicitly make the three-part type-token-occurrence distinction without introducing special terminology for the occurrence relation. Lyons (*1977*, Vol. 1, 13–18) has a section called *Type and token* which discusses the three-way distinction while using the same expressions for token and occurrence. However, the present terminology is familiar to logicians, as pointed out in my "Meanings of word: type-occurrence-token", *Bulletin of Symbolic Logic* 11(2005) 117.

At some point in the history of logic the type-token dichotomy gave way to the type-token-occurrence trichotomy. Given Peirce's penchant for trichotomies and his logical creativity, one is led to speculate, even hope, that it was Peirce who made this discovery.[34]

Another difference between sentence-tokens and sentence-types is that every sentence-token has a length in inches or centimeters, whereas the sentence-type, being abstract, has no length—not zero length, *no* length—

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[34]Substantially the same points have been made recently (Corcoran *2005*).

in inches or centimeters. Of course every expression-type has a "length" in character-occurrences. Even though the Nile is longer than the Niagara, in inches, 'the Nile' is "shorter" than 'the Niagara', in character-occurrences.

Moreover the sentence-token may be squeezed onto one line of a book or spread out over several lines, but these formattings do not apply to sentence-types. Probably the most important difference is that, as Peirce implies in the above passage, sentence-tokens are visible whereas sentence-types are invisible—a point rarely made. Church (*1956p*, 8) makes a closely related point without explicitly noting that they are invisible. From an ontological and epistemological point of view the difference between scientific treatment of string tokens and scientific treatment of string-types is dramatic. String tokens are studied in physics; string types are studied in mathematics—in the field known as string theory (Corcoran et al. *1974*, "String Theory", *J. Sym. Logic.* 39: 625–37).

Let us consider an application. Consider, e.g., the word 'letter'. In one sense there are exactly twenty-six *letters* (letter-types or ideal letters) in the English alphabet and there are exactly four letters in the word-type 'letter'. In another sense, there are exactly six *letters* (letter-repetitions or letter-occurrences) in the word-type 'letter'. In yet another sense, every new inscription (act of writing or printing) of 'letter' brings into existence six new *letters* (letter-tokens or ink-letters) and one new word that had not previously existed. The number of letter-occurrences (occurrences of a letter-type) in a given word-type is the same as the number of letter-tokens (tokens of a letter-type) in a single token of the given word. Many logicians fail to distinguish "token" from "occurrence" and a few actually confuse the two concepts. It is almost a rule that any article or book that explicitly mentions two of the three concepts without mentioning the third confuses the third with one of the other two.

There is a kind of cold war of words in the literature between two camps of logician philosophers, neither of which "officially" recognizes the type-token ambiguity of the word 'sentence'. Tarski may be taken as representing one camp, Quine the other. Whenever it is relevant for Tarski to clarify his use of the word 'sentence', he makes it a point to say that sentences are "inscriptions". He never mentions Peirce's work in this area. He almost never uses the word 'type' in Peirce's sense and he never uses the word 'token'. In the single place (*1930/1983*, 31) where he mentions "the type of a sentence", he says that it is "the set of all sentences [inscriptions]" in the same shape as the sentence [token][35]. This is not Peirce's view at all.

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[35] Although Tarski repeatedly says that shape is the determining factor, it is clear that he has not explored the details very far. Don't the left and right parentheses have the same shape? How about the plus and times signs, the less-than and greater than signs,

Tarski seems to imply repeatedly and flatly that it is "not strictly correct", an "error" or even a "widespread error", to use to the word 'sentence' in the sense of "sentence-type" (*1969/1993*, 114, *1930/1983*, 30, 31, *1933/1983*, 156).

Quine, however, in his last comprehensive statement of his philosophy of logic explicitly mentioned Peirce's work; and he used both 'type' and 'token' in Peirce's senses. But he never mentioned the type-token ambiguity of the word 'sentence': for Quine a sentence is a sentence-type-exactly the opposite of Tarski's usage. Let us look at Quine's own words[36] (*1970/1986*, 13–14): "In Peirce's terminology, utterances and inscriptions are *tokens* of the sentence or other linguistic expression concerned; and this [sentence or other] linguistic expression is the *type* of those utterances and inscriptions. In Frege's terminology, truth and falsity are the two *truth-values.* Succinctly, then, an eternal sentence is a sentence whose tokens all have the same truth-value [i.e. are all true or all false]."

In the recommended senses, both sentences and propositions are timeless. In contrast, each inscription or embodiment of a sentence comes into existence at a unique time and, in the fullness of time, will become illegible. Likewise, each thought of a proposition comes into existence as it is being thought of and goes out of existence as soon as the person thinking it turns to the next thought. Euclid and each of his translators thought of the Pythagorean Theorem, one and the same proposition which is timeless. But Euclid's thought of the Pythagorean Theorem was long defunct before his translators' thoughts of it came into being.

As I wrote this paper I reread an English translation of the Pythagorean Theorem and, despite its awkwardness and distracting ambiguity, I managed to have a thought of the Theorem. My thought was private; your belief that it ever existed is based on "mere hearsay". Some people are inclined to suppose that a particular thought of the Pythagorean Theorem is to the abstract proposition as a concrete inscription of a sentence expressing it is to the abstract sentence. There might be a sense in which this is true. But even so, it would be a stretch to conclude that a thought of a proposition is an embodiment of the proposition. A thought of a proposition seems to be an action, a kind of performance, more akin to an act of inscribing than to a static inscription.

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the vee and the caret, the zee and the en? Besides, there are arbitrarily "long" sentence-types but there is evidently a limit to the total number of character-tokens ever made in the history of writing. Of course, every argument against any given attempt to reduce types to tokens can be answered by an ever more elaborate "epicyclical construction".

[36]The italicizations are Quine's and the bracketed interpolations are mine.

# 8   Sentential Functions: Roles Sentences Play

> A proposition grasped by a person for the first time can be put in a form of words which will be understood by someone to whom the proposition is completely new. This would be impossible were we not able to distinguish parts in the proposition corresponding to the parts in the sentence, so that the structure of the sentence serves as a model for the structure of the proposition.—Frege *1979*, 1923.

The topic of sentential functions—the functions, roles, or uses of sentences—is one of the most interesting subjects in logic. A sentence can be mentioned. A sentence can be used to make a statement. A sentence can be used to make a promise, to make a request, to give an order, to refuse an order, to make an apology, to make a prediction, to ask a question, to insult, to inform, to deceive, to retract a previous statement. A sentence can be used ironically to make a statement that contradicts what would be "expected". Plato tells us that Cephalus, then quite old, "welcomed" the decline in his appetite. Continuing, Plato adds that Socrates "admired" Cephalus for saying so (*Republic* I, 329). A sentence can be used poetically or fictionally where the speaker is not responsible for the accuracy of what is expressed, but only for its poetic or literary qualities. The "pedagogical license" enjoyed by the teacher is warranted by the fact that the teacher's role demands combining the role of witness responsible for accuracy with the role of poet or storyteller responsible for something else (Corcoran *1999*).

In fact, of all the topics in logic suitable for discussion with an audience of logicians and non-logicians, this topic, despite its attractiveness, even its fascination, is hardly ever discussed. Consequently, the literature of logic and of non-logic abounds in confusions and fallacies that would be corrected by a small amount of concentrated attention to this topic.

The three-word sentence 'Ten is fifty', as peculiar as it may be, is in many ways typical. Of course, ten is not fifty. The number ten is only one-fifth of fifty. But, ten degrees Celsius equals fifty degrees Fahrenheit. The sentence 'Ten is fifty' can be used to express a true proposition concerning temperature conversions and to express a false proposition about numbers. It is exactly the same with corresponding sentences in other languages. We could just as easily consider the Portuguese sentence 'Dez é cinqüenta', the Spanish 'Diez es cincuenta', the German 'Zehn ist fünfzig', or the French 'Dix est cinquante'.

The English sentence mentioned above can also be used to express many other propositions, some true and some false; and it can be used to express

incoherent *messages*. Thus when someone has used the sentence to make an utterance in writing, we cannot, without further investigation, conclude that a true proposition or a false proposition was stated.

Examples of other propositions that can be expressed with this sentence can be discovered by imagining different contexts in which it might be written. The sentence 'ten is fifty' might be used to answer a question about the price of handkerchiefs; it might mean that the price of ten handkerchiefs is fifty cents, or fifty dollars, or fifty pounds, or fifty euros. In such cases the grammarian would explain that the three-word sentence is elliptical for an eight-word sentence by writing, e.g.: '[the price of] ten [handkerchiefs] is fifty [cents]'—using the brackets to indicate "restored ellipsis". It is only in a logically perfect language that grammatical structure of sentences models logical structure of the respective propositions expressed.

In order to determine the message, if any, that a given inscription of a sentence was intended to convey, it is often necessary to consider the context, including the discourse preceding the inscription in time and sometimes the past history of communication among the participants. Some interpret Frege as having puzzled his readers by saying that a word did not have meaning in isolation but only as part of a sentence. He asserted that the word gets its meaning from the sentence, and he denied that the sentence gets its meaning from the meanings of the words in it.[37] For years this was regarded as an extreme viewpoint.[38] Now we see to the contrary that Frege did not go far enough. Using the same exaggerated and elliptical style that Frege used, we can say that no sentence has meaning in isolation but only in the context of a communication—among several people, between two people, or by a single person, perhaps making a memorandum. We can affirm that determining what was meant by a sentence in a given inscription requires grasping the context and we can deny that it is sufficient to consider

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[37] In his *Grundlagen der Arithmetik* Frege wrote: "... we must always consider a complete sentence. Only in [the context of] the latter do words really have a meaning. ... It is enough if the sentence as a whole has a sense; by means of this its parts also receive their content."(Frege *1884/1964*, §60, tr. M. Mahoney). Later writers have followed him on this issue, not always giving due acknowledgement. Over a quarter century later Wittgenstein wrote: "Only sentences make sense; only in the context of a sentence does a name signify anything."(Wittgenstein *1921/1998*, §3.3 tr. D. Kolak).

[38] In view of the strangeness and implausibility of this view, not to mention the fact that neither Frege nor Wittgenstein gives a scintilla of evidence for it, we should not be surprised to find it being ignored by practicing linguists. For example, John Lyons does not mention it in his comprehensive, two-volume treatise *Semantics*. In fact, like many other semanticists, he takes a kind of diametrical opposite view to be obvious. He writes: "[It] ... is obvious enough ... that the meaning of a ... sentence is a product of the meanings of the words of which it is composed." (Lyons *1977*, Vol. I, 4). Also see the above quote from Frege *1923*.

only the sentence *per se*.[39]

Frege tells us "never to ask for the meaning of a word in isolation" (*1884/1959*, Intro., X). But he thinks that it is coherent to inquire as to the meaning of a sentence-type. If this paper has been successful, we have learned that we should never ask for the meaning or meanings of any expression-type, we should never ask "what does it mean to say such-and-such?" Rather, we should ask "what did so-and-so mean when writing such-and-such in thus-and-so text?"

## 9 Propositions Imply; Statements Implicate: Propositions Have Implications; Statements Have Implicatures

> The fact that a person defines an expression in a certain sense, in and of itself, is no evidence that the person uses the expression in that sense.—Frango Nabrasa, *2001*.

The proposition "Socrates taught Plato, who taught Aristotle" *implies* the proposition "Plato, who was taught by Socrates, taught Aristotle", which in turn implies the proposition "Plato taught Aristotle". But it does not imply the proposition "Socrates taught Aristotle". The relation in question is not transitive–even if it were, there would still be no implication.[40] Nor does it imply "Corcoran believes that Plato taught Aristotle".

Let us compare the above *proposition* with the *statement* I will now make.

Socrates taught Plato, who taught Aristotle.

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[39]Similar points have been made before by Quine, who says that the sentence per se is not what carries the truth or falsity but rather each token, more accurately each "event of utterance" (*1970/1986*, 13). Quine does not reveal it, but his view was anticipated by his teacher C. I. Lewis over thirty years earlier. Lewis wrote: "it is ... the word as read, the visual impression received, which arouses in the mind of the reader a corresponding meaning, and it is only when a mark ... gives rise to meaning that it is operating as a symbol" (*1932*, 311). Michael Scanlan observed that Lewis never follows up on or develops this interesting idea (personal communication). Other philosophers have made points similar to the one just attributed to Frege, Lewis, and Quine. But in many cases their works are so labored or so cryptic that it is hard to be confident that this point has even been grasped, let alone articulated. It is important to note that this thesis does not exclude the proposition that different tokens of the same sentence have shared significance.

[40]I am using 'implies' in the information-containment sense as elaborated in Corcoran *1998*; it is the second of twelve senses listed in Corcoran *1973*. In many cases, the information in a proposition involving a relational concept that determines a transitive relation does not contain the information that the relation is transitive. "One precedes two" does not imply "Every number preceding a given number precedes every number the given number precedes", which is logically equivalent to transitivity of number-theoretic precedence. Thinking otherwise is fallacious, perhaps the fallacy of premise-smuggling.

The last statement I made *implicates* "Corcoran believes that Socrates taught Plato, who taught Aristotle". Moreover, it implicates "Corcoran believes that Plato taught Aristotle". Every statement implicates every proposition implied by a proposition it implicates. It also implicates the proposition that I, Corcoran, understand the sentence / Socrates taught Plato, who taught Aristotle/. It also implicates that I believe that you understand the sentence.

As a rule, *propositions* about other people do not imply informative propositions about me. As a rule, statements implicate that the speaker believes the proposition stated, that the speaker understands the composite sentence uttered, and that the speaker believes that the audience understands it also.

The *implications* of a given proposition are the propositions whose information content is included in that of the given proposition.[41] Clearly, the information that Corcoran believes something is not contained in the information that Socrates taught Plato, who taught Aristotle. The *implicatures* of a given statement are the propositions it implicates; they include many of the propositions to which the speaker is morally committed by making the statement. Every proposition implies its *implications* and every statement implicates its *implicatures*. Propositions do not have implicatures.

According to the above conventions "imply" and "implicate" require non-human subjects: the former a proposition; the latter a statement. However, it is natural to used both elliptically so that human subjects are available. A person can be said to *imply* the propositions implied by a proposition the person stated; a person can be said to *implicate* a propositions if it is implicated by a statement the person made.[42]

It took years to figure out satisfactory ways to say what a proposition's implications are. We should not forget that both 'implication' and 'implicature' are ambiguous and that some of their meanings are vague. Maybe someone knows a satisfactory explanation of what a statement's implicatures are (Borchert *1996*, 225); I do not. Roughly, perhaps, we can say that a statement also implicates what the audience can take the statement as evidence for—assuming that the speaker is sincere, sane, responsible, etc.

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[41] This traditional view is elaborated in Corcoran *1998*.

[42] For these important conventions, which correspond to observed usage, I am indebted to the late Kenneth Barber (Barber and Corcoran *2009*). However, this terminology is not universally accepted. What I call a person's *implicatures* (singular *implicature*) others call a person's *implicata* (singular *implicatum*)—which is far too stuffy for my taste. Moreover, some restrict implicatures to persons only without allowing coherent mention of the implicatures of a statement, contrary to the usage found here and elsewhere (e. g., Borchert *1996*, 225). I thank my colleague David Braun for many helpful points including the need for this footnote.

A statement *of* a proposition is made *in* a language, *by* a speaker, *to* an audience, *at* a time and place. My statements today were all made in English *using* English sentences that can be used tomorrow to make other statements with other implicatures. A statement (*affirms / denies*) the propositions (implied / contradicted) by the proposition it states. If we define the *implications of a statement* to be the implications of the proposition it states, we can say that a statement affirms exactly what it implies and it denies exactly the propositions whose negations it implies. I do not recommend this definition which would burden the word 'implication' with yet another meaning having another range of applicability.

Normally, statements do not implicate all they affirm. My statement "Socrates taught Plato, who taught Aristotle" affirms but does not implicate the proposition "Plato taught Aristotle". Moreover, even when we can be confident that the speaker is being sincere, sane, responsible, etc., the statement need not be evidence for its affirmations. A statement of the axioms and definitions of arithmetic affirms the Goldbach Hypothesis or it affirms the Goldbach Hypothesis's negation.

A statement is *self-denying (SD)* if it *denies* one of its own *implicatures* (or equivalently, *implicates* a proposition it *denies*). Examples of SD propositions are: a statement to the effect that the speaker is not making a statement or is (using / not using) a word that in fact (is not / is) being used. A statement is *self-affirming (SA)* if it *affirms* one of its own *implicatures* (or, equivalently, it *implicates* a proposition it *affirms*). Examples of SA statements are: a statement to the effect that the speaker is making a statement or is (addressing / not addressing) an audience that in fact (is / is not) being addressed. A statement is (*correct / incorrect*) if (every / some) proposition it affirms or implicates is (true / false). Every SD statement is incorrect. Every statement of a contradiction is SD. Nevertheless, it is not the case that every SD statement affirms a contradiction: the proposition stated in a statement to effect that the speaker disbelieves a certain one of the statement's own affirmations need not contradict itself. As has been noted in discussions of "Moore's Paradox", such statements can be made using sentences such as 'Abe died but I *disbelieve* it'. Not every SA statement is correct. In fact, every SA lie is incorrect. A half-truth is a statement implicating a falsehood but affirming only truths.

## 10   Conclusion

> The proposition must be distinguished from
> the sentence, the combination of words or
> signs though which it is expressed; from the
> fact, the actual complex situation whose ex-
> istence renders it true or false; and from the
> judgment, which affirms or denies the propo-
> sition the proposition.—Eaton *1931*, 12.[43]

The five ambiguous words—sentence, proposition, judgment, statement, and fact—have received distinct recommended meanings that are needed for any sufficiently full discussion of logical phenomena such as self-denying and self-affirming statements. This paper has sketched a conceptual framework necessary for discussing issues that have concerned logicians over the years. I hope that most objective and informed logicians will agree that this framework is indispensable for comprehensive and comparative treatment of the history of logic. A historian inadvertently or willfully ignorant of any given one of these five concepts will be unable to give a full and fair treatment of a historical figure that uses it. I believe that appreciation of the place of this framework in logical thought will reveal how crude, evasive, and incomplete many important logic papers are. I hope that this awareness does not lessen the credit attributed to the authors of those works. If this paper succeeds, every future logic book should be affected.

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[43]Eaton seems to be combining the statement with the judgment. This is in keeping with other logicians who call Frege's "judgment-stroke" the "assertion sign" (Eaton *1931*, 369–374). But, the situation is actually much worse, this sign, which resembles a counterclockwise rotated tee, is often taken to indicate a logically cogent judgment—a judgment based entirely on "laws of logic", a logical inference.

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