

**SAMPLE SYLLABUS**

This document is published as an indication of the core content of the course. Instructors have responsibility of deciding on additional topics to be included, and the emphasis, ordering, and pacing of presentation.

Course Number: **MTH 309**

Course Title: **Introduction to Linear Algebra**

Credit Hours: **4**

Texts: **P. Selinger, Matrix Theory and Linear Algebra** ([www.mathstat.dal.ca/~selinger/linear-algebra/](http://www.mathstat.dal.ca/~selinger/linear-algebra/)),  
**B. Badzioch, MTH 309 Lecture Notes** ([github.com/bbadzioch/MTH309\\_F2019](https://github.com/bbadzioch/MTH309_F2019)).

Prerequisites: MTH 142

Notes: While this is a core course required for all math majors, usually over 70% of students taking MTH 309 are engineering majors. The course should cover several applications of linear algebra in natural sciences, engineering and computer science. Students taking this course need to be introduced to computer-based tools for performing linear algebra computation (row reduction, matrix multiplication, singular value decomposition etc.). Preferred tools for teaching this content are Python with its computing libraries (SymPy, NumPy) and Jupyter Notebook (or JupyterLab) as the programming environment. The Anaconda Distribution of Python ([www.anaconda.com/products/individual](http://www.anaconda.com/products/individual)) includes all these tools.

The table below indicates to what extent this course reflects each of the learning objectives of the undergraduate mathematics program. A description of learning objectives is available online at <http://www.buffalo.edu/cas/math/ug/undergraduate-programs.html>.

<b>Computational Skills:</b> extensively	<b>Analytical Skills:</b> moderately	<b>Practical Problem Solving:</b> moderately	<b>Research Skills:</b> little or not at all	<b>Communication Skills:</b> moderately
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Week	Topics
1	Systems of linear equations. Matrices and elementary row operations. Gauss-Jordan elimination. Pivot positions and pivot columns.
2	Applications of systems of linear equations (e.g. <i>balancing chemical equations, traffic flow in a network of streets, Google PageRank, polynomial interpolation</i> etc.). Vectors and vector equations.
3	Linear combinations. Span of a set of vectors. Linear independence. Matrix equations. The column space and the null space of a matrix.
4	Linear transformations. Matrix-vector multiplication. The standard matrix of a linear transformation.
5	Matrix algebra: addition, transpose, matrix multiplication. Properties of matrix algebra. Invertible matrices.
6	Applications of matrix algebra (e.g. <i>the Hill cipher, error correcting codes</i> etc.). <b>Midterm Exam I.</b>
7	Determinants. Determinants and elementary row operations, <i>cofactor expansion (optional), Cramer's rule (optional)</i> . Determinants and area and volume.
8	General vector spaces. Subspaces. Linear transformations of general vector spaces. Basis of a vector space and coordinates relative to a basis.
9	Change of basis. Dimension of a vector space. The rank theorem.
10	The dot product. Length of a vector, distance between vectors, orthogonality of vectors. Pythagorean Theorem. <b>Midterm Exam 2.</b>
11	Orthogonal and orthonormal bases. The Gram-Schmidt process. Orthogonal projections. Least square solutions of systems of linear equations. Application: fitting least square lines and curves.
12	General inner product spaces. Eigenvalues and eigenvectors. The characteristic polynomial of a matrix.
13	Matrix diagonalization. Orthogonal diagonalization and spectral decomposition of symmetric matrices.
14	Singular Value Decomposition. Applications of SVD (e.g. <i>image compression, data analysis</i> etc.).

Topics in *italics* are optional.

This is a sample schedule and may not match the schedule for your section. Instructors have responsibility of deciding which specific applications to cover, which optional topics to include or omit, additional topics to be included, and the emphasis, ordering, and pacing of presentation. For the schedule for your section consult the syllabus for your section or your instructor.

## Student Learning Outcomes for MTH 309 Introduction to Linear Algebra

**Assessment measures:** weekly homework assignments, 2 midterm exams, final exam. This is a sample syllabus. Your section may have more or fewer exams, more or fewer homework assignments, may utilize quizzes or other assessments not included in this sample, etc. For the detailed grading scheme for your section, consult the syllabus for your section or your instructor.

At the end of this course a student will be able to:	Assessment
<ul style="list-style-type: none"> <li>- represent systems of linear equations in vector and matrix form</li> <li>- determine if a system of equations is consistent and whether it has a unique solution</li> <li>- solve systems of linear equations using Gauss-Jordan elimination</li> </ul>	HW #2 Midterm 1 Final Exam
<ul style="list-style-type: none"> <li>- perform matrix-vector multiplication and understand how this operation defines a linear transformation between <math>\mathbb{R}^n</math> and <math>\mathbb{R}^m</math></li> <li>- add, multiply, and transpose matrices</li> <li>- determine whether a given matrix is invertible and compute its inverse if it exists</li> <li>- state and apply properties of matrix algebra</li> </ul>	HW #4, 5, 6 Midterm 1 Final Exam
<ul style="list-style-type: none"> <li>- compute determinants of matrices both by cofactor expansion and by row reduction</li> <li>- use Cramer's rule to solve systems of equations and to compute inverses of matrices</li> <li>- compute areas of parallelograms and volumes of parallelepipeds using determinants</li> <li>- understand relationship between the determinant of a matrix and properties of the linear transformation represented by the matrix</li> </ul>	HW #7 Midterm 2 Final Exam
<ul style="list-style-type: none"> <li>- recognize which sets of vectors of <math>\mathbb{R}^n</math> form a subspace</li> <li>- find a basis of the null space and the column space of a matrix</li> <li>- compute the rank of a matrix and the dimension of the column space of a matrix</li> <li>- compute bases of some vector spaces (null space, column space of a matrix etc.)</li> <li>- compute coordinates of a vector relative to a basis</li> <li>- compute the dimension of a vector space</li> <li>- compute dimensions of various subspaces defined by a matrix using the rank theorem</li> </ul>	HW #8, 9, 10 Midterm 2 Final Exam
<ul style="list-style-type: none"> <li>- understand the axiomatic definition of a vector space and know some examples of vector spaces other than <math>\mathbb{R}^n</math> (vector space of polynomials, vector space of matrices etc.)</li> <li>- recognize if a given function between vector spaces is a linear transformation</li> <li>- understand the notions of the kernel and the image of a linear transformation and their relationship to the null space and the column space of a matrix</li> </ul>	Midterm 2 Final Exam
<ul style="list-style-type: none"> <li>- compute the inner product of vectors in <math>\mathbb{R}^n</math></li> <li>- determine if a set of vectors in <math>\mathbb{R}^n</math> is orthogonal</li> <li>- compute the projection of a vector onto a subspace</li> <li>- orthogonalize a set of vectors using the Gram-Schmidt process</li> <li>- solve least squares problems</li> </ul>	HW #10, 11 Final Exam
<ul style="list-style-type: none"> <li>- compute the characteristic polynomial of a matrix, find eigenvalues and eigenvectors of the matrix</li> <li>- determine if a given matrix is diagonalizable and compute its diagonalization</li> <li>- use diagonalization of a matrix to compute its powers</li> <li>- compute orthogonal diagonalization of symmetric matrix</li> <li>- compute the Singular Value Decomposition of a matrix.</li> </ul>	HW #12 Final Exam
<ul style="list-style-type: none"> <li>- perform symbolic and numerical linear algebra computations using computer-based tools.</li> </ul>	Homework Assignments