THE SPRINGER CORRESPONDENCE AND RELATED TOPICS

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Let G be a reductive algebraic group with Lie algebra $\mathfrak{g}$. One example would be $G = SL_n(\mathbb{C}) = \{n \times n \text{ complex matrices with determinant } 1\}$ and its Lie algebra $\mathfrak{g} = \mathfrak{sl}_n(\mathbb{C}) = \{n \times n \text{ complex matrices with trace zero}\}$. There is a natural action of the algebraic group on its Lie algebra. For our matrix group example, this action is just conjugation. Let us consider the set $\mathscr{N} \subset \mathfrak{g}$ of nilpotent elements. This set forms an algebraic variety which splits into finitely many orbits under the action of G. These are called the nilpotent orbits of G. There is a finite group associated to each Lie algebra called the Weyl group. The Springer correspondence is a relationship between the nilpotent orbits of the Lie algebra and the irreducible representations of its Weyl group. The proof of this result relies on geometry, thus making it an early result in the area of Geometric Representation Theory.

In this talk, we will go over the definition of the Springer correspondence and see how perverse sheaves give a proof of it. We will also discuss how it relates to other topics in representation theory, and particularly emphasize its generalization due to Lusztig. I am currently working on a joint project with Martha Precup (Northwestern) and William Graham (UGA) that gives a new proof of the generalized Springer correspondence in the case of $G = SL_n(\mathbb{C})$, and I will talk briefly about this new construction. The talk is aimed to be accessible to an audience having taken a graduate level algebra course.

Date: February 6, 2017.