CSE 331: Algorithm and Complexity | Summer 2020

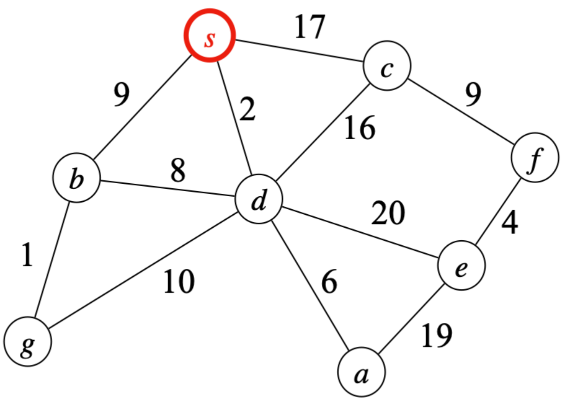
Homework 2

Instructor: Xiangyu Guo | **Deadline: 7/01/2020**

Your Name: ____________________ Your Student ID: ______________

Problems	1	2	3	4	Total
Max. Score	10	20	20	20	70
Your Score					

Problem 1 (10 points). Consider the following graph G with non-negative edge weights. Use Dijkstra's algorithm to compute the shortest paths from s to all other



vertices in G. Fill the following table to describe the execution of the algorithm. The algorithm maintains a set Γ of vertices, of which vertices have their $d(\cdot)$ value equals the shortest path length. The d value of a vertex $v \notin \Gamma$ is $\min_{u \in S:(u,v) \in E} (d(u) + w(u,v))$. The π value of a vertex v is the vertex $u \in \Gamma$ such that $d(v) = d(u) + w(u,v)$; if $d(v) = \infty$, then $\pi(v) =$"/".

iteration	vertex added to Γ	a		b		c		d		e		f		g	
		d	π	d	π	d	π	d	π	d	π	d	π	d	π
1	s	∞	/	9	s	17	s	2	s	∞	/	∞	/	∞	/
2															
3															
4															
5															
6															
7															
8															

Table 1: Dijkstra's algorithm for Shortest Path

Problem 2 (20 points). Assume we are given an undirected graph $G = (V, E)$ with non-negative edge weights $(w_e)_{e\in E}$, and two vertices s and t in V.

(2a) **(10 points)** Let T be the unique minimum spanning tree of G. Is the following statement true or false? If we change the weight of every edge e from w_e to w_e^2, then T is still the unique minimum spanning tree of G. Justify your answer.

(2b) **(10 points)** Let P be the unique shortest path from s to t. Is the following statement true or false? If we change the weight of every edge e from w_e to w_e^2, then P is still the unique shortest path from s to t. Justify your answer.

Problem 3 (20 points). Balanced strings are those who have equal quantity of "L" and "R" characters. Given a balanced string s, the goal is to split it into the *maximum amount* of balanced strings. For example, if s = "RLRRLLRLRL″, the optimal split is splitting into "RL", "RRLL", "RL", "RL", each substring contains same number of "L" and "R". Another example is s = "LRRLLRLR″, where there're multiple ways to split s, but the only optimal way is "LR"+"RL"+"LR"+"LR".

- **(5 points)** Suppose your greedy strategy is to pick the first few characters from s. Which characters are you going to choose?
- **(15 points)** Prove the safety property of your greedy strategy. (**Hint:** note that s itself is already balanced.)

Problem (20 points). Consider a long country road with houses scattered very sparsely along it. You may picture the road as a long straight line segment, with the starting point (mile stone 0) and the endpoint (mile stone L). Each house is identified by its distance to the western endpoint. A cell phone company wants to set up cell phone services along the road. The company can place a base station at any house. (The monthly charge will be waived if a base station is located in a house, so the house owners are eager to accommodate base stations). The power of base stations are limited that can only cover a distance of 5 miles. The goal for the company is to select a minimum number of base stations so that every house on the road is within 5 miles of a base station. (The bases stations are connected by other means, say by Satellite. So the distance between them can be more than 5 miles). See Figure 1 for a example.

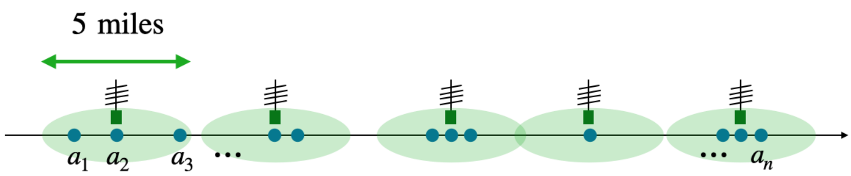


Figure 1: Using 5 base stations to cover all houses in A (denoted by the solid circles on the line).

A formal description of the problems: The input is an array A of n points: $A = a_1 < a_2 < \ldots < a_n$, where each a_i $(1 \leq i \leq n)$ represents a house. We need to select a subset $B \subseteq A$ such that: (1) for every point $a_i \in A$, there is a point $a_j \in B$ with

$|a_i - a_j| \leq 5$, and (2) the size of B is minimum, subject to condition (1). Describe a greedy algorithm for solving this problem. You need to prove the correctness of the algorithm.

- **(5 points)** Suppose your greedy strategy picks base station locations from left to right. Where would you set up your first base station?
- **(15 points)** Prove the safety property of your greedy strategy.