# Applied Cryptography and Computer Security CSE 664 Spring 2017 

Lecture 6: Digital Encryption Standard (DES)

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## Lecture Outline

- Previously we talked about:
- defining security for encryption
- using theoretical models for encryption
- In this lecture:
- move to practical constructions
- learn about design principles of block ciphers
- learn about how DES works


## Symmetric Encryption

- Block ciphers are used in practice to implement pseudorandom permutations
- it is desirable to base secure encryption on weaker assumptions than existence of pseudorandom permutations
- All block ciphers are heuristics
- no proofs of security
- Such algorithms are:
- normally very fast
- can be used as primitives in more complex cryptographic protocols


## Block Ciphers

- The algorithm maps an $n$-bit plaintext block to an $n$-bit ciphertext block
- Most modern block ciphers are product ciphers
- we sequentially apply more than one obfuscation technique to the message
- A common design for an algorithm is to proceed in iterations
- one iteration is called a round
- each round consists of similar operations
- iterations are used to amplify obfuscation


## Design Principles of Block Ciphers

- Specifying a random permutation on an $n$-bit block requires huge amount of storage
- there are $\left(2^{n}\right)$ ! permutations, each can be encoded in $\log \left(2^{n}!\right) \approx n 2^{n}$ bits
- how do we achieve similar effect with reasonable resources?
- Confusion-diffusion paradigm
- split a block into small chunks
- define a permutation on each chunk separately (confusion)
- mix outputs from different chunks by rearranging bits (diffusion)
- repeat to strengthen the result


## Design Principles of Block Ciphers

- Sulbstitution-permutation networks
- since a permutation on a block can be specified as a lookup table, this is called substitution
- instead of having substitutions defined by the key, such functions are fixed and applied to messages and keys
- mixing algorithm is called mixing permutation


## Design Principles of Block Ciphers



- For this type of algorithm to be reversible, each operation needs to be invertible


## Design Principles of Block Ciphers

- Let's denote one iteration or round by function $g$
- The initial state $s_{0}$ is the message $m$ itself
- In round $i$ :
- $g$ 's input is round key $k_{i}$ and state $s_{i-1}$
- $g$ 's output is state $s_{i}$
- The ciphertext $c$ is the final state $s_{N r}$, where $N r$ is the number of rounds
- Decryption algorithm applies $g^{-1}$ iteratively
- the order of round keys is reversed
- set $s_{N r}=c$, compute $s_{i-1}=g^{-1}\left(k_{i}, s_{i}\right)$


## Design Principles of Block Ciphers

- Another way to realize confusion-diffusion paradigm is through Feistel network
- in Feistel network each state is divided into halves of the same length: $L_{i}$ and $R_{i}$
- in one round:
- $L_{i}=R_{i-1}$
- $R_{i}=L_{i-1} \oplus f\left(k_{i}, R_{i-1}\right)$


## Design Principles of Block Ciphers



- Are there any advantages over the previous design?
- operations no longer need to be reversible, as the inverse of the algorithm is not used!
- reverse one round's computation as $R_{i-1}=L_{i}$ and $L_{i-1}=R_{i} \oplus f\left(k_{i}, R_{i-1}\right)$


## Design Principles of Block Ciphers

- In both types of networks, the substitution and permutation algorithms must be carefully designed
- choosing random substitution/permutation strategies leads to significantly weaker ciphers
- each bit difference in S-box input creates at least 2-bit difference in its output
- mixing permutation ensures that difference in one $S$-box propagates to at least 2 S-boxes in next round


## Block Ciphers

- Larger key size means greater security
- for $n$-bit keys, brute force search takes $2^{n} / 2$ time on average
- More rounds often provide better protection
- the number of rounds must be large enough for proper mixing
- Larger block size offers increased security
- security of a cipher also depends on the block length


## Data Encryption Standard (DES)

- In 1973 NIST published a solicitation for cryptosystems
- NIST stands for the National Institute of Standards and Technology
- previously the National Bureau of Standards
- was founded in 1901 within technology administration of the US Commerce Department
- develops and promotes standards and technology
- cryptographic standards are published in the Federal Information Processing Standards (FIPS)


## DES

- DES was developed by IBM as a modification of an earlier system Lucifer
- DES was adopted as a standard in 1977
- It was expected to be used as a standard for 10-15 years
- Was replaced only in 2001 with AES (Advanced Encryption Standard)
- DES characteristics:
- key size is 56 bits
- block size is 64 bits
- number of rounds is 16


## DES

- DES uses Feistel network
- Feistel network is used in many block ciphers such as DES, RC5, etc.
- not used in AES
- in DES, each $L_{i}$ and $R_{i}$ is $\mathbf{3 2}$ bits long; $k_{i}$ is 48 bits long



## DES

- DES has a fixed initial permutation $I P$ prior to 16 rounds of encryption
- The inverse permutation $I P^{-1}$ is applied at the end



## DES

- The $f$ function $f\left(k_{i}, R_{i-1}\right)$

1. first expands $R_{i-1}$ from 32 to 48 bits ( $k_{i}$ is $\mathbf{4 8}$ bits long)
2. XORs expanded $R_{i-1}$ with $k_{i}$
3. applies substitution to the result using $\mathbf{S}$-boxes
4. and finally permutes the value

## DES $f$ Function



## DES

- There are 8 S-boxes
- S-boxes are the only non-linear elements in DES design
- they are crucial for the security of the cipher
- Example: $S_{1}$

| $\mathbf{1 4}$ | $\mathbf{4}$ | $\mathbf{1 3}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1 5}$ | $\mathbf{1 1}$ | $\mathbf{8}$ | $\mathbf{3}$ | $\mathbf{1 0}$ | $\mathbf{6}$ | $\mathbf{1 2}$ | $\mathbf{5}$ | $\mathbf{9}$ | $\mathbf{0}$ | $\mathbf{7}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbf{0}$ | $\mathbf{1 5}$ | $\mathbf{7}$ | $\mathbf{4}$ | $\mathbf{1 4}$ | $\mathbf{2}$ | $\mathbf{1 3}$ | $\mathbf{1}$ | $\mathbf{1 0}$ | $\mathbf{6}$ | $\mathbf{1 2}$ | $\mathbf{1 1}$ | $\mathbf{9}$ | $\mathbf{5}$ | $\mathbf{3}$ | $\mathbf{8}$ |
| $\mathbf{4}$ | $\mathbf{1}$ | $\mathbf{1 4}$ | $\mathbf{8}$ | $\mathbf{1 3}$ | $\mathbf{6}$ | $\mathbf{2}$ | $\mathbf{1 1}$ | $\mathbf{1 5}$ | $\mathbf{1 2}$ | $\mathbf{9}$ | $\mathbf{7}$ | $\mathbf{3}$ | $\mathbf{1 0}$ | $\mathbf{5}$ | $\mathbf{0}$ |
| $\mathbf{1 5}$ | $\mathbf{1 2}$ | $\mathbf{8}$ | $\mathbf{2}$ | $\mathbf{4}$ | $\mathbf{9}$ | $\mathbf{1}$ | $\mathbf{7}$ | $\mathbf{5}$ | $\mathbf{1 1}$ | $\mathbf{3}$ | $\mathbf{1 4}$ | $\mathbf{1 0}$ | $\mathbf{0}$ | $\mathbf{6}$ | $\mathbf{1 3}$ |

- input to each S-box is 6 bits $b_{1} b_{2} b_{3} b_{4} b_{5} b_{6}$
- row $=b_{1} b_{6}$, column $=b_{2} b_{3} b_{4} b_{5}$
- output is 4 bits


## DES

- More about S-boxes..
- a modified version of IBM's proposal was accepted as the standard
- some of the design choices of S-boxes weren't public, which triggered criticism
- in late 1980s - early 1990s differential cryptanalysis techniques were discovered
- it was then revealed that DES S-boxes were designed to prevent such attacks
- such cryptanalysis techniques were known almost 20 years before they were discovered by others


## DES Key Schedule

- Key computation consists of:
- circular shift
- permutation
- contraction



## DES

- Why does decryption work?
- round function $g$ is invertible
- compute $L_{i-1}=R_{i} \oplus f\left(k_{i}, L_{i}\right)$
- compute $R_{i-1}=L_{i}$
- in the beginning apply $I P$ and at the end apply $I P^{-1}$
- round keys $k_{16}, \ldots, k_{1}$ and the $f$ function are computed as before


## Attacks on DES

- The key size is $\mathbf{5 6}$ bits, is this too small?
- Is the design non-linear enough to be hard to break?
- Are there cryptanalysis techniques that we can use against DES?
- two techniques exist: linear and differential cryptanalysis


## DES Weak Keys

- The master key $k$ is used to generate 16 round keys
- Some keys result in the same round key to be generated in more than one round
- this reduces complexity of the cipher
- Solution: check for weak keys at key generation
- DES has 4 weak keys:
- 00000000000000
- 0000000 FFFFFFF
- FFFFFFF 0000000
- FFFFFFF FFFFFFF


## Attacks on DES

- Brute force attack
- try all possible $2^{56}$ keys
- time-consuming, but no storage requirements
- It was conjectured in 1970s that a cracker machine could be built for \$20 million
- In 1990s RSA Laboratories called several DES challenges
- Challenge II-2 was announced in 1998
- the winner was Electronic Frontier Foundation (EFF)
- they built a DES Cracker machine for less than $\mathbf{\$ 2 5 0 , 0 0 0}$
- it found the key in 56 hours and searched 88 billion keys per second


## Attacks on DES

- RSA Laboratories called Challenge III in 1999
- was solved in a record time
- cooperative effort of the DES Cracker and a worldwide network of 100,000 computers
- the key was found in 22 hours 15 minutes
- over 245 billion keys were tested per second
- http://www.distributed.net/des


## Attacks on DES

- Differential Cryptanalysis
- complex technique that tracks the behavior of pairs of text blocks evolving along each round
- set $\Delta_{m}=m_{1} \oplus m_{2}$ and $\Delta_{c}=c_{1} \oplus c_{2}$
- distribution of $\Delta_{c}$ 's, given $\Delta_{m}$ 's, may reveal information about the key
- Not effective against DES
- attack on 8-round DES requires $2^{38}$ known plaintext-ciphertext pairs
- attack on 16-round DES requires $2^{47}$ chosen plaintext pairs


## Attacks on DES

- Linear Cryptanalysis
- a slightly more recent technique (1993) based on finding linear approximations to describe DES transformations
- the goal is to find some bits of the key
- How does DES do?
- the attack has no practical implication on the cipher
- attack on 8-round DES requires $2^{21}$ known plaintext pairs
- attack on 16-round DES requires $2^{43}$ known plaintext pairs


## Increasing Security of DES

- The best attack against DES is brute force search
- DES uses a 56-bit key and this raised concerns
- One proposed solution is double DES
- apply DES twice by using two different keys $k_{1}$ and $k_{2}$
- encryption $c=E_{k_{2}}\left(E_{k_{1}}(m)\right)$
- decryption $m=D_{k_{1}}\left(D_{k_{2}}(c)\right)$
- The resulting key is $2 \cdot 56=112$ bits, so it is more secure
- is it really?


## Meet-in-the-Middle Attack

- The goal of the attack is, given pairs $(m, c)$, find keys $k_{1}$ and $k_{2}$
- It is based on the observation that

$$
c=E_{k_{2}}\left(E_{k_{1}}(m)\right) \text { and } E_{k_{1}}(m)=D_{k_{2}}(c)
$$

- Thus, the idea is to try all possible $2^{56}$ keys for $k_{1}$ and all possible keys for $k_{2}$ until a match is found


## Meet-in-the-Middle Attack

- $c=E_{k_{2}}\left(E_{k_{1}}(m)\right)$ and $E_{k_{1}}(m)=D_{k_{2}}(c)$
- Algorithm steps:
- encrypt $m$ with all possible $2^{56}$ keys $k_{1}$
- store all pairs $\left(k_{1}, E_{k_{1}}(m)\right)$ sorted by $E_{k_{1}}(m)$
- decrypt $c$ with all possible $2^{56}$ keys $k_{2}$
- for each decrypted result $D_{k_{2}}(c)$ check to see if there is a match $D_{k_{2}}(c)=E_{k_{1}}(m)$
- when a match is found, verify the keys on another pair ( $m^{\prime}, c^{\prime}$ )
- if the second pair matched, accept the keys $k_{1}$ and $k_{2}$
- The overall effort is on the order of $2^{56}$


## Meet-in-the-Middle Attack

- Why do we need the second pair?
- block size is 64 bits, so for a given $m$ there are $2^{64}$ potential ciphertexts
- with two 56-bit keys, there are $2^{112}$ potential double keys that can $\operatorname{map} m$ to $c$
- for a single pair $(m, c)$ the number of double keys $\left(k_{1}, k_{2}\right)$ that produce $c=E_{k_{2}}\left(E_{k_{2}}(m)\right)$ is $2^{112} / 2^{64}=2^{48}$
- thus $2^{48}$ false alarms for a single pair are expected
- with one more pair ( $m^{\prime}, c^{\prime}$ ), extra 64 bits of known text, the alarm rate goes down to $2^{48} / 2^{64}=1 / 2^{16}$


## Meet-in-the-Middle Attack

- With two pairs $(m, c)$ and $\left(m^{\prime}, c^{\prime}\right)$ the correct keys $k_{1}$ and $k_{2}$ can be determined with probability $1-1 / 2^{16}$
- Known plaintext attack against double DES succeeds in about $2^{56}$ work as opposed to $2^{55}$ on average for DES
- the 112-bit key provides the level of security similar to that of the 56-bit key
- Is there any hope to make DES stronger?


## Triple DES

- Triple DES with two keys $k_{1}$ and $k_{2}$ :
- encryption $c=E_{k_{1}}\left(D_{k_{2}}\left(E_{k_{1}}(m)\right)\right)$
- decryption $m=D_{k_{1}}\left(E_{k_{2}}\left(D_{k_{1}}(c)\right)\right)$
- Key space is $2 \cdot 56=112$ bits
- There is no know practical attack against 3DES with 2 keys
- e.g., Merkle and Hellman attack requires $2^{56}$ chosen plaintext-ciphertext pairs and $2^{56}$ work


## Triple DES

- Triple DES with three keys $k_{1}, k_{2}$, and $k_{3}$ :
- encryption $c=E_{k_{3}}\left(D_{k_{2}}\left(E_{k_{1}}(m)\right)\right)$
- decryption $m=D_{k_{1}}\left(E_{k_{2}}\left(D_{k_{3}}(c)\right)\right)$
- Key space is $3 \cdot 56=168$ bits
- There is no known practical attack against it either
- Many applications that used DES switched to 3DES
- Can be made backward compatible by setting $k_{1}=k_{2}$ or $k_{3}=k_{2}$


## Summary of Attacks on DES

- DES
- best attack: brute force search
$-2^{55}$ work on average
- no other requirements
- Double DES
- best attack: meet-in-the-middle
- requires 2 plaintext-ciphertext pairs
- requires $2^{56}$ space and about $2^{56}$ work
- Triple DES
- best practical attack: brute force search


## Summary

- DES is a block cipher with
- key size of 56 bits
- block size of 64 bits
- Feistel structure
- and 16 rounds
- DES was the de facto standard for over 20 years
- The best attack against DES is brute force search
- Triple DES can be used to improve resistance to such attacks

