MTH142 - Fall 2014

Quiz - 9

Last Name:

First Name:

Person #:

Problem: Determine whether the following series is **absolutely convergent, conditionally convergent, or divergent**.

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{n+1}.$$

Justify your answer using appropriate test(s).

This is an alternating series, so we would like to apply the Alternating Series Test. To do this, we need to check whether the sequence $a_n = \frac{1}{n+1}$ is decreasing and $\lim_{n\to\infty} a_n = 0$. Since $n+1 < n+2$ for any $n \geq 1$, $a_{n+1} = \frac{1}{n+2} < \frac{1}{n+1} = a_n$ for any $n \geq 1$. This proves that a_n is decreasing. That $\lim_{n\to\infty} \frac{1}{n+1} = 0$ is clear. Hence, by the Alternating Series Test, the above series is convergent. Meanwhile the series

$$\sum_{n=1}^{\infty}\left|(-1)^n\frac{1}{n+1}\right| = \sum_{n=1}^{\infty}\frac{1}{n+1}$$

is divergent by, for example, the Limit Comparison Test since

$$\lim_{n\to\infty}\frac{\frac{1}{n+1}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n}{n+1} = 1.$$

As a result, the given series is **not** absolutely convergent, it is conditionally convergent.