MTH142 - FALL 2014

QUIZ - 7

Last Name:

First Name:

Person #:

Problem: Consider the sequence

$$a_n = \frac{n}{3^n} \text{ for } n \geq 1.$$

(a) (3 pts) Show that the sequence is decreasing.

You may show this in two different ways:

- Consider the function $f(x) = \frac{x}{3^x}$. Since $f'(x) = \frac{3^x - x3^x \ln 3}{(3^x)^2} = \frac{1 - x\ln 3}{3^x} < 0$ for any $x \geq 1$, the function $f(x)$, and hence $f(n) = a_n$, is decreasing.
- Note that

$$\begin{aligned} n+1 &< 3n \\ \frac{n+1}{3} &< n \\ \frac{n+1}{3^{n+1}} = \frac{n+1}{3 \cdot 3^n} &< \frac{n}{3^n} \\ a_{n+1} &< a_n \end{aligned}$$

for any $n \geq 1$.

(b) (3 pts) Show that the sequence is bounded.

Since the sequence is decreasing by part (a), $a_1 \leq a_n$ for all $n \geq 1$.

(c) (4 pts) Find the limit of the sequence. Justify your answer.

By the Monotone Convergence Theorem, the sequence has a limit. To find the limit, consider the function $f(x) = \frac{x}{3^x}$. Since $\lim_{x\to\infty} f(x) = \lim_{n\to\infty} a_n$, and

$$\begin{aligned} \lim_{x\to\infty} f(x) &= \lim_{x\to\infty} \frac{x}{3^x} \\ &= \lim_{x\to\infty} \frac{1}{3^x \ln 3} \\ &= 0, \end{aligned}$$

by L'Hopital's Rule, we have $\lim_{n\to\infty} a_n = 0$