

MTH142 - FALL 2014

QUIZ - 5

Last Name:

First Name:

Person #:

Problem: Evaluate the area of the surface obtained by rotating the graph of the function

$$f(x) = 2\sqrt{x}$$

for $x \in [3, 8]$ about the x -axis. (**Show your work.**)

Solution: The formula for the surface area in this case is

$$\int_3^8 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx.$$

We compute that $f'(x) = \frac{1}{\sqrt{x}}$, and hence

$$\begin{aligned} \int_3^8 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx &= \int_3^8 2\pi 2\sqrt{x} \left(\sqrt{1 + \left[\frac{1}{\sqrt{x}}\right]^2} \right) dx \\ &= \int_3^8 4\pi \left(\sqrt{x} \sqrt{1 + \frac{1}{x}} \right) dx \\ &= \int_3^8 4\pi \left(\sqrt{x \left(1 + \frac{1}{x}\right)} \right) dx \\ &= \int_3^8 4\pi (\sqrt{x+1}) dx \\ &= 4\pi \frac{2}{3} (\sqrt{x+1})^3 \Big|_3^8 \\ &= 4\pi \frac{2}{3} (9^{3/2} - 4^{3/2}) \\ &= 4\pi \frac{2}{3} (27 - 8) \\ &= \frac{152\pi}{3}. \end{aligned}$$