Last Name:

First Name:

Person #:

Problem:

$$\int_2^\infty \frac{1}{(x-1)^2}dx.$$

(a) Explain why the above integral is improper.

This is an improper integral because the integral is taken over an interval of **infinite** length.

(b) Determine whether the above improper integral is **convergent or divergent**. If it is convergent, evaluate it.

In order to determine whether the above improper integral is convergent or divergent check if the following limit exists:

$$\lim_{t\to\infty}\int_2^t \frac{1}{(x-1)^2}dx.$$

$$\lim_{t\to\infty}\int_2^t \frac{1}{(x-1)^2}dx \underset{(u=x-1)}{=} \lim_{t\to\infty}\int_1^{t-1}\frac{1}{u^2}du$$

$$= \lim_{t\to\infty}(-\frac{1}{u})|_1^{t-1}$$

$$= \lim_{t\to\infty}[-\frac{1}{t-1}+1]$$

$$\underset{(\lim_{t\to\infty}-\frac{1}{t-1}=0)}{=} 1.$$

Hence the improper integral is convergent with value 1.