MTH142 - Fall 2014

Quiz - 3

Last Name:

First Name:

Person #:

Problem: Compute the integral

$$\int \frac{(\ln x)^2}{x^2} dx.$$

(Show your work clearly)

Solution: Use integration by parts with

- $u = (\ln x)^2 \implies du = 2\frac{\ln x}{x} dx.$
- $dv = \frac{1}{x^2} dx \implies v = -\frac{1}{x}.$

(Logarithmic function trumps algebraic function!) Then the IBP formula

$$\int udv = uv - \int vdu$$

gives

$$\begin{aligned} \int \frac{(\ln x)^2}{x^2} dx &= -(\ln x)^2 \cdot \frac{1}{x} - \int -\frac{1}{x} \cdot 2\frac{\ln x}{x} dx \\ &= -\frac{(\ln x)^2}{x} + 2\int \frac{\ln x}{x^2} dx. \end{aligned}$$

To compute the rightmost integral in the bottom row, we use integration by parts with

- $u = \ln x \implies du = \frac{1}{x} dx.$
- $dv = \frac{1}{x^2} dx \implies v = -\frac{1}{x}.$

Then

$$\begin{aligned} \int \frac{\ln x}{x^2} dx &= -\ln x \cdot \frac{1}{x} - \int -\frac{1}{x} \cdot \frac{1}{x} dx \\ &= -\frac{\ln x}{x} + \int \frac{1}{x^2} dx \\ &= -\frac{\ln x}{x} - \frac{1}{x}. \end{aligned}$$

As a result,

$$\int \frac{(\ln x)^2}{x^2} dx = -\frac{(\ln x)^2}{x} - \frac{2\ln x}{x} - \frac{2}{x} + C.$$