MTH142 - FALL 2014

QUIZ - 2

Last Name:

First Name:

Problem: A heavy rope, 50 ft long, weighing 40 lbs hangs over the edge of a building 100 ft high.

(a) (5 pts) Approximate the work required to pull the rope to the top of the building by a Riemann sum.

(b) (5 pts) Express the work as an integral and evaluate it.

Solution:

(a) Start by subdividing the length of the rope, which is the longest distance any point on the rope can travel, into n equal subintervals:

$$0 = x_0 < x_1 < \cdots < x_{n-1} < x_n = 50,$$

where $\Delta x = \frac{50}{n}$ and $x_i = i\frac{50}{n}$. For each $i = 1, \ldots, n$, choose x_i^* in $[x_{i-1}, x_i]$. Since the weight density of the rope is $\frac{40}{50} = 0.8$ lbs/ft, the weight of the ith portion of the rope with length Δx is $0.8\Delta x$ lbs. Approximating the distance traveled (against gravitational force) by the ith portion of the rope via x_i^*, we find that the work done in pulling the ith portion to the top is approximated by

$$W_i \simeq (0.8\Delta x) \cdot x_i^*.$$

Hence an approximation to the total work done is given by

$$W \simeq \sum_{i=1}^{n} 0.8 x_i^* \Delta x.$$

(b) To find the actual work done, take limit as $n \to \infty$ to get a definite integral out of the Riemann sum found in part (a), and evaluate the integral:

$$\begin{aligned} \int_0^{50} 0.8x dx &= 0.8\frac{x^2}{2}|_0^{50} \\ &= 0.4[50^2 - 0] \\ &= 1000\ ft \cdot lb \end{aligned}$$