

## MTH142 - FALL 2014

## QUIZ - 1

**Last Name:**

**First Name:**

**Problem:** Let  $f(x) = x^2$  and  $g(x) = x + 2$  be two functions.

- (a) (4 pts) Illustrate the area bounded by the graphs of the functions  $f$  and  $g$ .
- (b) (6 pts) Compute the area between the graphs of  $f$  and  $g$ .

**Solution:**

- (b) Note that the graphs of  $f$  and  $g$  intersect at points with  $x$ -coordinates satisfying the equation

$$x^2 = x + 2.$$

In other words,  $x^2 - x - 2 = 0$ . Using the quadratic formula, we find that the intersection points have  $x$ -coordinates  $x = -1$  and  $x = 2$ . To find the area between the curves we compute the integral

$$\int_{-1}^2 |x^2 - (x + 2)| dx.$$

Note that  $x^2 - (x + 2) = x^2 - x - 2 \leq 0$  on  $[-1, 2]$  since the leading coefficient of  $x^2 - x - 2$  is  $1 > 0$ . Therefore, the above integral becomes

$$\int_{-1}^2 (x + 2 - x^2) dx.$$

Evaluating this integral we find the area:

$$\begin{aligned} \int_{-1}^2 (x + 2 - x^2) dx &= \left[ \frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2 \\ &= \left[ \frac{4}{2} + 4 - \frac{8}{3} \right] - \left[ \frac{1}{2} - 2 + \frac{1}{3} \right] \\ &= \frac{9}{2}. \end{aligned}$$