MTH 636 - Spring 2014
SYLLABUS

Contact information

Cagatay Kutluhan

E-mail: kutluhan@buffalo.edu

Office: Math 117

Office hours: TR 4:00 - 5:00 pm, or by appointment

Lectures

Time: TR 12:30 - 1:50 pm

Place: Math 250

Course description: The aim of this course is to establish some of the mathematical background underlying the interactions between differential geometry and topology, and to form the foundation required to move on to studying gauge theories. In physics, gauge theories are used in formulating models to explain physical phenomena, but they also exhibit strong connections to topology and have been a hot topic of research in geometry and topology for decades. Topics to be covered include vector bundles, principal bundles, connections, and characteristic classes. A more detailed outline of topics to be covered and a list of references are below.

Prerequisitities: Knowledge of differential and algebraic topology, namely, topics covered by the geometry and topology portion of the second qualifying exam, is required.

Grading: Your letter grade for the course will be based on regular homework assignments.

Topics:

(1) Tangent and cotangent bundles of differentiable manifolds, tensor and exterior bundles, Lie derivative of tensor fields and exterior derivative of forms, integration on differentiable manifolds, de Rham cohomology.
(2) Real and complex vector bundles, homomorphisms between vector bundles, orientations, constructing new vector bundles out of old ones: subbundles, quotient bundles, dual vector bundles, homomorphism bundles, direct sum of vector bundles, tensor and exterior product of vector bundles; pull-back of vector bundles, classifying spaces and universal vector bundles.
(3) Connections on vector bundles, parallel transport, holonomy, and curvature. Geometric structures on vector bundles and compatible connections.
(4) Lie groups, vector fields on Lie groups, Lie algebras and the exponential map, adjoint and coadjoint representations, Maurer–Cartan form.

(5) Principal bundles, associated bundles, reduction of structure group, Ehressman connection, principal connections, parallel transport, holonomy, and curvature, space of principal connections. Ambrose–Singer theorem.
(6) Characteristic classes, invariant polynomials, and Chern–Weil theory.

References

1. William M. Boothby, *Introduction to differentiable and Riemannian manifolds*, Pure and Applied Mathematics, 120. Academic Press, Inc., Orlando, FL, 1986. xvi+430 pp.
2. Raoul Bott and Loring W. Tu, *Differential forms in algebraic topology*, Graduate Texts in Mathematics, vol. 82, Springer-Verlag, New York, 1982.
3. S. S. Chern, W. H. Chen, and K. S. Lam, *Lectures on differential geometry*, Series on University Mathematics, vol. 1, World Scientific Publishing Co. Inc., River Edge, NJ, 1999.
4. Dale Husemoller, *Fibre bundles*, Second edition. Graduate Texts in Mathematics, vol. 20. Springer-Verlag, New York-Heidelberg, 1975.
5. John M. Lee, *Introduction to smooth manifolds*, Graduate Texts in Mathematics, vol. 218, Springer, New York, 2013.
6. Shigeyuki Morita, *Geometry of differential forms*, Translations of Mathematical Monographs, vol. 201. Iwanami Series in Modern Mathematics. American Mathematical Society, Providence, RI, 2001.
7. John W. Milnor and James D. Stasheff, *Characteristic classes*, Princeton University Press, Princeton, N. J., 1974, Annals of Mathematics Studies, No. 76.