

Practice Problems for the Second Examination (revised March 25, 2008)

The second exam will cover all of the material since the first exam. On the exam, I will ask you to do some of the following.

- (a) Symbolize English sentences into MPL.
- (b) Prove validity and invalidity of various schemas with respect to various classes of standard models.
- (c) Provide some (relatively simple) proofs that various wffs or schemas are theorems in the various systems of modal logic that were discussed in class, e.g., system *KTB*.

To prepare for the exam, I suggest that you do the following.

- (a) Review your notes and your copy of the lecture notes.
- (b) Be sure that you can do all of the problems on all of the homeworks. Read the solutions to the homeworks, and make sure that you understand them.
- (c) Practice on some of the problems below.

I. Symbolizations

- 1. It's possible for John to get what he wants.
- 2. If two is even, then it is necessary that two is even.
- 3. If Beth knows what Jerry is up to, then she can stop him.
- 4. It is a contingent truth that Wilma is a banker.
- 5. If Wilma is a banker, then that is only contingently the case.
- 6. Snoopy's being a dog entails that he is a dog.
- 7. Sue's being a used car salesman is compatible with her being honest.
- 8. If Julie passed the exam, then she must have known the material well.
- 9. Elwood can get the job only by bribing the personnel officer.
- 10. Frank could not have failed to go to the hospital.
- 11. It's possible for John to fail to be a plumber, but if he is a plumber then he must have at least one hand.
- 12. Frank's being sick and poor is sufficient for him to get Medicaid.

II. Leibniz Validity Prove that all instances of the following schemas as Leibniz-valid.

- 1. $\Diamond\Box\phi \rightarrow \Box\phi$
- 2. $\Diamond\Box\phi \rightarrow \Diamond\phi$
- 3. $\Diamond\phi \rightarrow \Box\Box\Diamond\phi$
- 4. $\Box(\Diamond\phi \rightarrow \Diamond\Diamond\phi)$

III. Leibniz-Invalidity Prove that each of the following schemas has instances that are Leibniz-invalid.

1. $\diamond\phi \rightarrow \phi$
2. $\diamond\phi \rightarrow \Box\phi$
3. $\Box\diamond\phi \rightarrow \phi$
4. $\Box(\phi \vee \psi) \rightarrow (\Box\phi \vee \Box\psi)$
5. ~~$\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$~~ (Note on March 25, 2008: This is schema K, which is clearly valid w.r.t. the class of Leibnizian models.)

IV. Leibniz Validity and Invalidity For each of the following, decide whether all instances are valid, or whether some are invalid, and prove it.

1. $\Box\diamond\phi \rightarrow \diamond\phi$
2. $\diamond\Box\phi \rightarrow \Box\diamond\phi$
3. $\Box\diamond\phi \rightarrow \diamond\Box\phi$
4. $(\sim\diamond\phi \wedge \psi) \rightarrow (\sim\phi \wedge \psi)$

V. Validity in Classes of Standard Models Show that all instances of the following schemas are valid in the corresponding class of standard models.

1. $(\Box\phi \wedge \Box\psi) \rightarrow \Box(\phi \wedge \psi)$ *any class*
2. $\diamond(\phi \vee \psi) \rightarrow (\diamond\phi \vee \diamond\psi)$ *any class*
3. $\diamond\phi \vee \diamond\sim\phi$ *serial*
4. $\diamond(\phi \vee \sim\phi)$ *serial*
5. $\Box(\phi \rightarrow \Box\psi) \rightarrow (\diamond\phi \rightarrow \psi)$ *symmetric*
6. $(\Box\phi \vee \Box\psi) \rightarrow \Box(\Box\phi \vee \Box\psi)$ *transitive*
7. $\Box(\diamond\phi \wedge \diamond\psi) \rightarrow \diamond(\diamond\phi \wedge \psi)$ *euclidean*
8. $\diamond\Box\phi \rightarrow \Box\diamond\phi$ *equivalence*

VI. Invalidity

1. Prove that there are instances of schemas 4 and 5 that are invalid with respect to the class of reflexive symmetric standard models.
2. Prove that there are instances of the schemas B and 5 that are invalid with respect to the class of reflexive transitive standard models.
3. Prove that there are instances of the schemas T and B that are invalid with respect to the class of serial transitive standard models.
4. Prove that there are instances of the schema D that are invalid with respect to the class of symmetric transitive standard models.
5. Prove that there are instances of the schema 4 that are invalid with respect to the class of serial euclidean standard models.
6. Prove that there are instances of the schema T that are invalid with respect to the class of serial symmetric standard models.

VII. Theorems Prove that all instances of the following schemas are theorems of the indicated system of modal logic. You may write any theorem of PL as a line. If one line follows from another by PL, you may write the line into the derivation without giving the intermediate steps (annotation, 'PL'). You may use the rules RM, RN, RE, and RR. You may use the theorem

$\Box\phi \leftrightarrow \sim\Diamond\sim\phi$.

1. $\vdash_K (\phi \rightarrow \sim\sim\phi)$
2. $\vdash_K \Box\phi \rightarrow \Box(\psi \rightarrow \phi)$
3. $\vdash_K \Box(\phi \wedge \psi) \leftrightarrow (\Box\phi \wedge \Box\psi)$
4. $\vdash_K \Diamond(\phi \vee \psi) \leftrightarrow (\Diamond\phi \vee \Diamond\psi)$
5. $\vdash_{KD} \Box\Box\phi \rightarrow \Diamond\Diamond\phi$
6. $\vdash_{KD} \sim\Box(\phi \wedge \sim\phi)$
7. $\vdash_{KT} \Box\Box(\phi \rightarrow \psi) \rightarrow (\Diamond\phi \rightarrow \psi)$
8. $\vdash_{KTB} \Box(\phi \rightarrow \Box\psi) \rightarrow (\Diamond\phi \rightarrow \psi)$
9. $\vdash_{KT4} \Box\phi \rightarrow \Box\Box\phi$
10. $\vdash_{KT4} \Diamond\Diamond\Box\phi \rightarrow \Diamond\phi$
11. $\vdash_{KT45} \Box(\Box\phi \leftrightarrow \Box\Box\phi)$
12. $\vdash_{KT5} (\Diamond\phi \wedge \Diamond\psi) \rightarrow \Diamond(\Diamond\phi \wedge \Diamond\psi)$