Philosophy 214/414
Logical Methods in Philosophy
Spring 2008

Practice Problems for Exam 1

I. Well-formed formulas and grammatical trees

For each of the expressions below, say whether (a) it is a wff of PL, (b) it is an abbreviation of a wff of PL, or (c) it is neither of these. If (a), provide a grammatical tree. If (b), give its unabbreviated form and give a tree for that. If (c), say *briefly* why it is not a wff.

1. $\sim\sim(P\rightarrow T_{73})$.
2. $(\sim M_{21}\rightarrow Z)$
3. $Q\vdash_{PL}Q$.
4. $(S\rightarrow Q)\wedge(T\rightarrow\sim P)$
5. $(\varphi\rightarrow(\psi\rightarrow\chi))\rightarrow((\varphi\rightarrow\psi)\rightarrow(\varphi\rightarrow\chi))$
6. $P\in\{P, Q\}$

II. Validity and Semantic Consequence

7. Prove that for all wffs φ and ψ, $\vDash_{PL} (\sim\varphi\rightarrow\sim\psi)\rightarrow(\psi\rightarrow\varphi)$
(This theorem is part of the proof of the Soundness Theorem. A truth-table may appear as part of your proof, but you must explain why it helps show the above theorem.)
8. Prove that it is *not* the case that for all wffs φ and ψ, $\vDash_{PL} (\varphi\rightarrow\psi)\rightarrow (\sim\varphi\rightarrow\sim\psi)$
(The most straightforward way to prove this is to take an instance and show that it is false in some assignment. Please fully specify the assignment [specify the truth-value that it assigns to every sentence letter].)
9. Prove Theorem 2.12 (a)-(h) on p. 34.
10. Prove that it is *not* the case that for all sets of wffs Γ and Δ, if Γ is satisfiable and Δ is satisfiable, then $\Gamma\cup\Delta$ is satisfiable.
(Hint: The most straightforward proof specifies two satisfiable sets, and proves that no assignment can satisfy the union of them.
11. Prove that for all wffs φ, if φ is unsatisfiable, then $\vDash_{PL}\sim\varphi$.
12. Prove that for all wffs φ and ψ, and all sets of wffs Γ, if either $\Gamma \vDash_{PL}\sim\varphi$ or $\Gamma \vDash_{PL}\psi$, then $\Gamma\vDash_{PL}(\varphi \rightarrow\psi)$.
(Hint: Use *separation of cases*, which is the following inference rule: "From (If **A** then **C**) and (if **B** then **C**), infer (If either **A** or **B**, then **C**)." To apply it here, first state that you will use separation of cases. Then prove that if $\Gamma \vDash_{PL}\sim\varphi$, then $\Gamma\vDash_{PL}(\varphi \rightarrow\psi)$. Next prove that if $\Gamma \vDash_{PL}\psi$, then $\Gamma\vDash_{PL}(\varphi \rightarrow\psi)$. Then infer, by separation of cases, that if *either* $\Gamma \vDash_{PL}\sim\varphi$ *or* $\Gamma \vDash_{PL}\psi$, then $\Gamma\vDash_{PL}(\varphi \rightarrow\psi)$.)

III. Theorems and Syntactic Consequence

13. Prove that $\sim\varphi \vdash_{PL} (\varphi \rightarrow \psi)$. Do not use the Deduction Theorem.
14. Prove that $(R \rightarrow P) \rightarrow (T \rightarrow S) \vdash_{PL} (\sim P \rightarrow \sim R) \rightarrow (T \rightarrow S)$. You may use the Deduction Theorem.
15. Prove Theorem 2.18 (a)-(f), p. 43 in the book.
16. Prove that for all wffs φ and ψ, and all sets of wffs Γ and Δ, if $\Gamma \vdash_{PL} \varphi$ and $\Delta \vdash_{PL} (\varphi \rightarrow \psi)$, then $\Gamma \cup \Delta \vdash_{PL} \psi$.
17. Prove that it is *not* the case that for all wffs φ and all sets of wffs Γ, if $\Gamma \cup \{\varphi\}$ is inconsistent, then $\Gamma \cup \{\sim\varphi\}$ is consistent.
    (Hint: find an instance, and prove that it is an instance.)
18. Prove that for all wffs φ and ψ, and sets of wffs $\Gamma$and Δ, if $\Gamma \vdash_{PL} (\sim\varphi \rightarrow \sim\psi)$ and $\Delta \vdash_{PL} \psi$, then $\Gamma \cup \Delta \vdash_{PL} \varphi$.