

Homework Assignment #7

Due at the beginning of class on Wednesday, March 5

I. Propositions and entailment

1. Prove that for all wffs ϕ and ψ , and all Leibnizian models M , $|(\phi \vee \psi)|^M = |\phi|^M \cup |\psi|^M$. (Hint: to prove this, you need to prove that for all $w \in W$, $w \in |(\phi \vee \psi)|^M$ iff $w \in |\phi|^M \cup |\psi|^M$).
2. Prove that for all wffs ϕ and ψ , and all Leibnizian models M , the proposition expressed by $(\phi \wedge \sim \phi)$ in M M -entails the proposition expressed by ψ in M . (Hint: the empty set is a subset of every set.)
3. Prove that for all wffs ϕ and ψ , and all Leibnizian models M , if $\models_{Lz}(\phi \leftrightarrow \psi)$, then the proposition expressed by ϕ in M is identical with the proposition expressed by ψ in M (that is, $|(\phi|^M = |\psi|^M)$).

II. Symbolization

Follow **all** of the instructions.

(a) Symbolize the following sentences into the language of MPL. (Write the original English sentence on your answer sheet before you symbolize.) Feel free to use any of the primitive and defined sentential and modal connectives: \sim , \rightarrow , \wedge , \vee , \leftrightarrow , \diamond , \square . If a symbolization scheme is provided below, use it. Otherwise, create your own and *be sure to present your symbolization scheme prominently*. **If the sentence is ambiguous** (can be symbolized in two or more non-equivalent ways), give the **two** most plausible symbolizations, and indicate with a '*' which (if either) is more likely to be true. Use $\square(\phi \rightarrow \psi)$ rather than $(\phi \Rightarrow \psi)$ or $(\phi \text{ fish-hook } \psi)$

(b) For each sentence, state an interpretation of the modality (e.g., nomological or deontic) that a reasonable author is likely to have intended.

4. Clinton's winning is incompatible with McCain's winning. (P : Clinton wins. Q : McCain wins.)
5. Mary must not be a brain in a vat, if she can see her hands.
6. If God knows that I will stand up, then I cannot fail to stand up.
7. Gore couldn't have failed to lose the election.
8. Adam's existing entails that he is human.
9. Sue's being a Democrat is compatible with her being a fiscal conservative.
10. It's possible that this table might not have been made of wood.
11. Descartes could exist and yet not be spatially extended, but his body could not exist and not be spatially extended. (P : Descartes exists. Q : Descartes is extended. R : Descartes's body exists. S: Descartes's body is spatially extended.)
12. John's acting freely is incompatible with his action's being determined by prior events. (P: John acts freely. Q: John's actions are determined by prior events.)
13. If it's possible that God exists, then it is necessarily possible that He exists.

IV. Symbolization and Validity

Here is a traditional definition of ‘God’.

DEFINITION x is *God* iff x is the omnipotent, omniscient, perfectly good, necessarily existing being who created the universe.

The following seems to follow from the definition.

For all x (x is identical with God iff x is the omnipotent, omniscient, perfectly good, necessarily existing being who created the universe.)

Moreover, since this follows from a definition, it should be a necessary truth.

Necessarily: for all x (x is identical with God iff x is the omnipotent, omniscient, perfectly good, necessarily existing being who created the universe.)

This claim seems to entail the following claim.

Necessarily: God exists iff God necessarily exists.

This justifies the first premise of the following proof of the existence of God. The second premise is justified by the fact that we can conceive that God exists.

1. It is necessary that: God exists iff God necessarily exists.
 2. It is possible that God exists.
 3. Therefore, God exists.
14. Do the following.
- (a) Symbolize this argument. Be sure to display your symbolization scheme prominently.
 - (b) Determine whether the resulting symbolic argument is Leibniz-valid. If it is, prove that it is. If it is not, prove that it is not. (Clarification: suppose that ϕ , ψ , and χ are your symbolizations of (1), (2), and (3), respectively. Then determine whether the following is true: $\{\phi, \psi\} \models_{LZ} \chi$. When doing this, keep in mind our definition of $\Gamma \models_{LZ} \phi$.)
 - (c) Required for graduate students, optional for undergraduates: present one plausible objection to the premises of the argument.

V. Relations

15. Prove that for all sets W and all binary relations R on W , if R is symmetric and transitive on W , then R is Euclidean on W .
16. Prove that for all sets W and all binary relations R on W , if R is symmetric and Euclidean on W , then R is transitive on W .
17. Prove that it is *not* the case that for all sets W and all binary relations R on W , if R is an equivalence relation on W , then R is a total relation on W .
18. Prove that it is *not* the case that for all sets W and all binary relations R on W , if R is a symmetric relation on W then R is reflexive on W .

IV. Validity Show that all instances of the following schemas are valid with respect to *any* class of standard models. You may assume the derived truth conditions for connectives for standard models.

19. $\Box(\phi \wedge \psi) \rightarrow (\Box\phi \wedge \Box\psi)$