

Homework Assignment #11

Due at the beginning of class on Wednesday, April 9

I. Symbolizations Symbolize the following English sentences into the language of Temporal Propositional Logic (as well as possible). Feel free to use any of the primitive and defined sentential and temporal connectives: $\sim, \rightarrow, \wedge, \vee, \leftrightarrow, \mathbf{G}, \mathbf{H}, \mathbf{F}, \mathbf{P}$. Please write the English sentence before symbolizing. If a symbolization scheme is provided, use it. Otherwise, provide your own and display it prominently. If you think that the English sentence is ambiguous, provide the two most plausible symbolizations.

1. Carol is a student, but she won't always be one.
2. There have always been poor people, and there always will be.
(p : There are poor people.)
3. John sang and Mary danced.
4. John sang while Mary danced.
5. If a sea battle will occur, then it has always been the case that a sea battle will occur.
(p : A sea battle occurs.)

II. Tense Logic

6. Prove that for all classes T of temporal standard models, and all TPL wffs ϕ , $\models_T \phi \rightarrow \mathbf{GP}\phi$.
7. Prove that if D is the class of dense temporal standard models (the class of temporal standard models in which $<$ is dense), then: for all wffs ϕ of TPL, $\models_D \mathbf{P}\phi \rightarrow \mathbf{PP}\phi$.
8. Prove that it is *not* the case that for all classes T of temporal standard models, and all TPL wffs ϕ , $\models_T \phi \rightarrow \mathbf{FP}\phi$. (Note: contrast this with problem #6 above.)

III. Validity Prove the following theorems. You may assume the derived truth conditions for $\wedge, \vee, \leftrightarrow$, and \diamond .

9. For all CMPL wffs ϕ and ψ , $\models_{SC} \Box\psi \rightarrow (\phi \Box \rightarrow \psi)$.
10. For all CMPL wffs ϕ and ψ , $\models_{SC} (\phi \wedge \psi) \rightarrow (\phi \Box \rightarrow \psi)$.
11. For all CMPL wffs ϕ and ψ , $\models_{SC} (\phi \Box \rightarrow \psi) \vee (\phi \Box \rightarrow \sim\psi)$.
(*Hint*: note that $(\phi \Box \rightarrow \psi) \vee (\phi \Box \rightarrow \sim\psi)$ is true at a world w in a model M iff: if $\sim(\phi \Box \rightarrow \psi)$ is true at w in M , then $(\phi \Box \rightarrow \sim\psi)$ is true at w in M . So showing that the latter is true at a world in a model is sufficient for showing that the former is true at a world in a model. Using this fact may make the proof easier.)

Continued . . .

IV. Invalidity Prove the following theorems. (Consider an instance of the schema; describe a type of model in which the instance is false at some world of the model; prove that the instance is false at that world in that model.) You may assume the derived truth conditions for \wedge , \vee , \leftrightarrow , and \diamond .

12. It is not the case that for all CMPL wffs ϕ and ψ ,
 $\models_{\text{SC}}(\phi \Box \rightarrow \psi) \rightarrow (\sim \psi \Box \rightarrow \sim \phi)$.
13. It is not the case that for all CMPL wffs ϕ , ψ , and χ :
 $\models_{\text{SC}} ((\phi \Box \rightarrow \psi) \wedge (\psi \Box \rightarrow \chi)) \rightarrow (\phi \Box \rightarrow \chi)$.