Philosophy 214/414
Logical Methods in Philosophy
Spring 2008

Homework 1

Due at the beginning of class, Wednesday, January 23. (There may be additions or deletions, which will be announced in class.)

A. Well Formed Formulas For each of the following expressions, state whether or not it is a wff of PL. For each expression that is not a wff, say *briefly* why it is not. (You do *not* need to prove that a non-wff is not a wff.) For each expression that is a wff, provide a grammatical tree.

1. $T_{7,344,907}$
2. $\sim(\sim P\rightarrow X\rightarrow\sim\sim Y))$
3. $(\sim(\sim\sim Q))$
4. $(\psi\rightarrow(\varphi\rightarrow\varphi))$
5. $(P\rightarrow\sim(\sim Q\rightarrow R))$
6. $(P\rightarrow(Q\rightarrow R)$

B. More on Wffs Prove the following metatheorem.

7. '$(P\sim Q)$' is not a wff of PL.

C. Abbreviations Each of the following expressions is an abbreviation of an official wff of PL. Convert each to an official wff by adding missing parentheses, substituting definitions for abbreviations, and so on.

8. $P\rightarrow Q$
9. $\sim P\wedge(Q\wedge R)$
10. $(Q\vee R)\leftrightarrow\sim S$

D. Formally Defined Languages Which of the following languages is formally defined? Say briefly why or why not.

11. Language X
    Primitive Vocabulary: '1 ', '2 ', '3 ', '4 '
    Well-Formed Formulas: Any sequence of vocabulary items such that their sum is greater than 5. (That is: if *a*, *b*, . . ., *n* are primitive vocabulary items, then
    ab...n
    is a wff iff $a + b + \ldots + n > 5$.) There are no other wffs.

12. Language Y
    Primitive Vocabulary: ‘|’
    Well-formed Formulas: all and only those sequences of primitive vocabulary items such that the number of items in the sequence is even.

13. Language Z
    Primitive Vocabulary: ‘e’, ‘i’, ‘l’, ‘s’, ‘v’
    Well-formed Formulas: ‘elvis lives’

<u>E. Quotation</u> In class, we often drop quotation marks. In the exercise below, I want you to use them completely correctly. Starting with the following expressions, produce true sentences of our metalanguage (English plus some technical vocabulary) by *either* (i) adding regular quotation marks, *or* (ii) adding corner quotes, *or* (iii) displaying one or more of the expressions, *or* (iv) deleting regular quotes or corner quotes, *or* (v) doing more than one of the preceding *or* (v) making no changes at all.

14. Bill Clinton contains bowels.
15. Bill Clinton contains vowels.
16. If ~~R is a wff of PL, then so is ~R.
17. If ‘~~φ’ is a wff of PL, then so is ~φ.
18. If φ, ψ, and χ are wffs of PL, then so is ((φ→ψ)→χ).

<u>F. More Grammar</u>

19. Let φ be a wff of PL. Let *s*(φ) be the number of occurrences of sentence letters in φ. Let *b*(φ) be the number of occurrences of *binary* connectives in φ (namely the number of occurrences of ‘→’ in φ). Example: if φ = ‘(P→(P→Q))’, then *s*(φ)=3 and *b*(φ)=2. Prove that for all φ, *s*(φ) = *b*(φ) + 1. (In words: the number of sentence-letter occurrences in a wff φ is equal to the number of occurrences of binary connectives in φ plus one.)
    Hint: use induction on the complexity of wffs in PL.