

The loss-averse newsvendor problem[☆]

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Abstract

Newsvendor models are widely used in the literature, and usually based upon the assumption of risk neutrality. This paper uses loss aversion to model manager's decision-making behavior in the single-period newsvendor problem. We find that if shortage cost is not negligible, then a loss-averse newsvendor may order more than a risk-neutral newsvendor. We also find that the loss-averse newsvendor's optimal order quantity may increase in wholesale price and decrease in retail price, which can never occur in the risk-neutral newsvendor model.

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1. Introduction

The newsvendor problem is one of the fundamental models in stochastic inventory theory [1]. The newsvendor's objective is to choose an optimal order quantity to balance his cost (or disutility) of ordering too many against his cost (or disutility) of ordering too few. Because of its simple but elegant structure, the newsvendor problem also applies in various settings such as capacity planning, yield management, insurance, and supply chain contracts.

The standard newsvendor model is based upon risk neutrality so that managers will select an order quantity to maximize expected profit. However, in practice,

there are many counter examples indicating that managers' order quantity decisions are not always consistent with maximizing expected profit. For example, Kahn [2] finds that Chrysler held larger inventory stocks than competitors such as GM and Ford before early 1980, which seemed to imply the *stockout-avoidance* behavior (see also [3,4]). We refer to the deviation of the newsvendor's optimal order quantity from the profit maximization order quantity as *decision bias* in the newsvendor problem.

One major reason why decision bias may exist in the newsvendor problem is that a manager may have preferences other than risk neutrality [5]. In this paper, we attempt to use an alternative choice model—loss aversion—to describe the newsvendor decision bias. Loss aversion states that people are more averse to losses than they are attracted to same-sized gains. There are two main reasons why we use loss aversion to describe the decision bias in the newsvendor problem. First, loss aversion is both intuitively appealing and

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well supported in finance, economics, marketing, and organizational behavior (see [6,7] for more detailed reviews). This paper is especially motivated by two empirical studies of managerial decision-making under uncertainty. The first study by MacCrimmon and Wehrung [8] is based on questionnaire responses from 509 high-level executives in American and Canadian firms, and interviews with 128 of those executives in 1973–1974. The second study by Shapira [9] is based on interviews with 50 American and Israeli executives in 1984–1985. Managers' decision-making behavior in both studies is consistent with loss aversion. Second, in contrast with the widely applications and empirical supports of loss aversion in other fields, unfortunately, the development of loss aversion to describe manager's newsvendor decision bias is still in its early stages. As far as we know, Schweitzer and Cachon [5] is the only paper studying a loss-averse newsvendor problem. They show that a loss-averse newsvendor (with no shortage cost) will order strictly less than a risk-neutral newsvendor. However, the newsvendor's decision-making behavior is unclear from their model if (1) shortage cost is considered and (2) some price or cost is changing. To fill in this research gap, we extend their model by considering the shortage cost and summarizing comparative statics of price and cost changes.

The purpose of our research is to build a theoretical model that characterizes the outcomes of the decision bias of a loss-averse newsvendor, i.e., to show when, how, and why loss aversion causes the decision bias in the newsvendor problem. We use a simple "kinked" piecewise-linear loss-aversion utility function to study the single-period newsvendor model. We find: (1) a loss-averse newsvendor will order less than the risk-neutral newsvendor if he faces low shortage cost and the more loss-averse, the less his optimal order quantity; (2) a newsvendor will order more than the risk-neutral newsvendor if he faces high shortage cost and the more loss-averse, the more his optimal order quantity. We also discuss some comparative statics of price and cost changes. We find that the loss-averse newsvendor's optimal order quantity may increase in wholesale price and decrease in retail price, which can never occur in the risk-neutral newsvendor model. Our research contributes to the newsvendor literature in two main aspects. First, we quantify the newsvendor decision bias and show how the newsvendor loss aversion decision bias interacts with the business conditions, e.g., retail price, wholesale price, shortage cost, and degree of loss aversion, *when shortage cost exists*. Second, since a loss-averse newsvendor orders from his supplier a different quantity from the profit-maximizing quantity, the

total supply chain performance is suboptimal. Hence, our research findings also lend insights into how loss aversion contributes to supply chain inefficiency and may lead to new policies for mitigating these effects.

This paper is organized as follows. In Section 2, we briefly review the literature related to our research. In Section 3, we analyze our single-period loss-averse newsvendor model. In Section 4, we analyze the effects of changing parameter values. Finally, in Section 5, we draw our conclusions and identify opportunities for future research.

2. Related newsvendor literature

The traditional newsvendor model is based upon risk neutrality; managers place orders to maximize expected profits (e.g., [1,10,11]). Recently, a number of attempts have been made to extend our understanding of the newsvendor model. Lippman and McCardle [12] study the competitive newsvendor problem. Petruzzi and Dada [13] provide a survey of the newsvendor problem where both price and quantity are set simultaneously. Casimir [14,15] study the value of information in the single and multi-item newsvendor problems. Carr and Lovejoy [16] consider a newsvendor who chooses which customers to serve given his fixed capacity. Dana and Petruzzi [17] study a newsvendor model in which consumers choose between attempting to purchase the newsvendor's product and an exogenous alternative. Van Mieghem and Rudi [18] introduce newsvendor networks, which generalize the classic newsvendor model and allows for multiple products and multiple processing and storage points. Cachon and Kok [19] study a newsvendor model with clearance pricing for the leftover inventory at the end of the selling season, which contrasts with the traditional assumption of a constant salvage value assigned to each unit of unsold inventory. Mostard et al. [20] and Mostard and Teunter [21] study the newsvendor problem where undamaged returned products from customers during the selling season are still resalable before the season ends.

In addition to risk neutrality, some researchers have attempted to use risk aversion within Expected Utility Theory (EUT) to describe the decision-making behavior in the newsvendor problem. Eeckhoudt et al. [22] study a risk-averse newsvendor who is allowed to obtain additional orders if demand is higher than his initial order. They find that a risk-averse newsvendor will order strictly less than a risk-neutral newsvendor. Agrawal and Seshadri [23] investigate a risk-averse and price-setting newsvendor problem. They find that a risk-averse newsvendor will charge a higher price and order less

than the risk-neutral newsvendor if the demand distribution has the *multiplicative* form of relationship with price. Also, the risk-averse newsvendor will charge a lower price if the demand distribution has the *additive* form of relationship with price, but the effect on the quantity ordered depends on the demand sensitivity to selling price.

Another stream of literature closely related to our paper deals with loss aversion. There have since been many economic field tests and applications of loss aversion in financial markets (e.g., [24]), life savings and consumptions (e.g., [25]), labor supply (e.g., [26]), marketing (e.g., [27]), real estate (e.g., [28]), and organizational behavior (e.g., [29]).

3. The loss-averse newsvendor model

Consider a newsvendor selling short-life-cycle products with stochastic demand. At the beginning of the selling season, the newsvendor orders Q products at a wholesale price w' from a supplier and sells at a retail price $p' > w'$ during the selling season. Demand X is a nonnegative random variable with a probability density function (PDF) $f(x)$ and a cumulative distribution function (CDF) $F(x)$ defined over the continuous interval I . To simplify notation, we assume without loss of generality that $\inf I = 0$ (e.g., if $\inf I > 0$, then Q represents the amount to increase the order beyond the minimum possible demand). If realized demand x is higher than Q , then unit shortage cost penalty s is incurred on $x - Q$ units. The shortage cost s may account for the cost of an emergency delivery and/or goodwill cost due to the effect of a stockout on future sales and profits. If realized demand x is lower than Q , then the newsvendor salvages $Q - x$ unsold products at a unit value $v' < w'$. We note that v' may be negative, in which case it may represent a disposal cost. As with most of the traditional newsvendor models, we assume $F(x)$ is continuous, differentiable, invertible, and strictly increasing over I . We let $\bar{F}(x) = 1 - F(x)$ denote the tail distribution. The newsvendor has the following payoff function:

$$\pi(x, Q) = \begin{cases} \pi_-(x, Q) \\ = (p' - v')x - (w' - v')Q & \text{if } x \leq Q, \\ \pi_+(x, Q) \\ = (p' - v')Q - s(x - Q) \\ - (w' - v')Q & \text{if } x > Q. \end{cases} \quad (1)$$

Without loss of generality, we standardize the profit expression by defining $p = p' - v'$, $w = w' - v'$, and selecting a unit of currency such that $p = 1$. Thus, the

payoff function can be rewritten as

$$\pi(x, Q) = \begin{cases} \pi_-(x, Q) = x - wQ & \text{if } x \leq Q, \\ \pi_+(x, Q) \\ = Q - s(x - Q) - wQ & \text{if } x > Q, \end{cases} \quad (2)$$

where $s > w - 1$ and $w \in (0, 1)$.

Lemma 1. *For any $Q \in I$, let $q_1(Q) = wQ$ and $q_2(Q) = (1 + s - w)Q/s$. If $s \geq (1 - w)Q/(\sup I - Q)$, then the newsvendor has two breakeven quantities of realized demand, $q_1(Q)$ and $q_2(Q)$ where if $x < q_1(Q)$ or $x > q_2(Q)$, the newsvendor's realized profit is negative and if $q_1(Q) < x < q_2(Q)$, the newsvendor's realized profit is positive. If $s < (1 - w)Q/(\sup I - Q)$, then the newsvendor only has one break-even quantity $q_1(Q)$ where the newsvendor's realized profit is negative if and only if $x < q_1(Q)$.*

Lemma 1 shows that if realized demand relative to Q is relatively low, i.e., $x < q_1(Q)$, then the newsvendor will face *overage loss*; if shortage cost s and realized demand relative to Q are relatively high, i.e., $s > (1 - w)Q/(\sup I - Q)$ and $x > q_2(Q)$, then the newsvendor will face *underage loss*; and if realized demand is between $q_1(Q)$ and $q_2(Q)$, then the newsvendor will face gains. Note that if demand distribution interval I is unbounded, e.g., normal and exponential distributions, then $q_2(Q) < \sup I = \infty$, and the newsvendor always faces both overage and underage losses. However, if I is bounded (e.g., uniform distribution) and shortage cost s is relatively small, i.e., $s < (1 - w)Q/(\sup I - Q)$, then $q_2(Q) > \sup I$ and the newsvendor only faces overage loss but does not face underage loss.

Let W_0 denote the newsvendor's reference level (e.g., his initial wealth) at the beginning of the selling season. We consider a simple piecewise-linear form of loss aversion utility function

$$U(W) = \begin{cases} W - W_0, & W \geq W_0, \\ \lambda(W - W_0), & W < W_0, \end{cases} \quad (3)$$

where $\lambda \geq 1$ is defined as the loss aversion coefficient. Therefore, there exists a kink at the reference level W_0 if $\lambda > 1$, and higher values of λ imply higher levels of loss aversion. Without loss of generality, we normalize the newsvendor's reference level to zero, i.e., $W_0 = 0$. This piecewise-linear form of loss aversion utility function in Fig. 1 has been widely used in the economics, finance, and operations management literature (see e.g., [5,30,31]), and is an approximation of the non-linear (and hence leading to intractability of the model) utility functions.

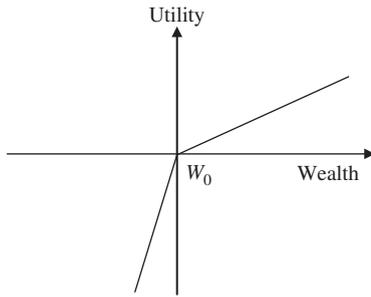


Fig. 1. The piecewise-linear loss aversion utility function.

After mapping the newsvendor’s payoff function (2) into his utility function (3), we can express the newsvendor’s expected utility $E[U(\pi(X, Q))]$ as

$$E[U(\pi(X, Q))] = E[\pi(X, Q)] + (\lambda - 1) \left(\int_0^{q_1(Q)} \pi_-(x, Q) f(x) dx + \int_{q_2(Q)}^{\sup I} \pi_+(x, Q) f(x) dx \right). \tag{4}$$

The loss-averse newsvendor’s expected utility is the expected profit plus the total expected underage and overage losses, biased by a factor of $\lambda - 1$. If $\lambda = 1$, then the newsvendor is risk neutral and the second term in (4) drops out.

Theorem 1. $E[U(\pi(X, Q))]$ is concave for all Q in the range of $F(x)$ and there exists a unique optimal order quantity Q_λ^* that maximizes the expected utility and satisfies the following first-order condition:

$$(1 + s - w)\bar{F}(Q_\lambda^*) - wF(Q_\lambda^*) + (\lambda - 1) \times [(1 + s - w)\bar{F}(q_2(Q_\lambda^*)) - wF(q_1(Q_\lambda^*))] = 0. \tag{5}$$

If the newsvendor is risk-neutral, i.e., $\lambda = 1$, then the first-order condition (5) reduces to

$$(1 + s - w)\bar{F}(Q_1^*) - wF(Q_1^*) = 0, \tag{6}$$

and we get

$$Q_1^* = F^{-1} \left(\frac{1 + s - w}{1 + s} \right).$$

Define $(1 + s - w)\bar{F}(q_2(Q))$ in (5) as the loss-averse newsvendor’s marginal underage loss and $wF(q_1(Q))$ as his marginal overage loss. Our next theorem shows how loss aversion contributes to decision bias, i.e., the difference between the risk-neutral and the loss-averse newsvendor’s optimal order quantities.

Theorem 2. For any $\lambda > 1$, if $(1 + s - w)\bar{F}(q_2(Q_1^*)) > wF(q_1(Q_1^*))$, then $Q_\lambda^* > Q_1^*$ and $dQ_\lambda^*/d\lambda > 0$; if $(1 + s - w)\bar{F}(q_2(Q_1^*)) = wF(q_1(Q_1^*))$, then $Q_\lambda^* = Q_1^*$ and $dQ_\lambda^*/d\lambda = 0$; otherwise, $Q_\lambda^* < Q_1^*$ and $dQ_\lambda^*/d\lambda < 0$.

Theorem 2 identifies a necessary and sufficient condition under which the loss-averse newsvendor will order more than (i.e., positive bias), equal to, or less than (i.e., negative bias) the risk-neutral newsvendor. More specifically, it shows that at the risk-neutral profit-maximizing order quantity: (1) if the loss-averse newsvendor’s marginal underage loss is equal to his marginal overage loss, then no matter how loss-averse he is, he will order the same as the risk-neutral newsvendor; (2) if the loss-averse newsvendor’s marginal overage loss is larger than his marginal underage loss, then the more loss-averse, the greater his reduction in order quantity relative to the risk-neutral newsvendor; and (3) if the loss-averse newsvendor’s marginal underage loss is larger than his marginal overage loss, then the more loss-averse, the greater his increase in order quantity relative to the risk-neutral newsvendor. Result (2) is similar to a result in Schweitzer and Cachon [5] who analyze the case of zero shortage cost and find that the loss-averse newsvendor will always order less than the risk-neutral newsvendor. The following corollary shows that this conclusion continues to hold for positive shortage cost up to a threshold value.

Corollary 1. For any $\lambda > 1$, there exists a shortage cost $s' > 0$ such that for all $s < s'$, $Q_\lambda^* < Q_1^*$ and $dQ_\lambda^*/d\lambda < 0$.

The following results help characterize the relationship between Q_λ^* and Q_1^* , given $\lambda > 1$ and $s > 0$, for three probability distributions. Let

$$g(s, w) = (1 + s - w)\bar{F}(q_2(Q_1^*)) - wF(q_1(Q_1^*)), \tag{7}$$

and note that the sign of $Q_\lambda^* - Q_1^*$ is determined by the sign of $g(s, w)$, i.e., $dE[U(\pi(X, Q_1^*))]/dQ = (\lambda - 1)g(s, w)$. The function $g(s, w)$ expressed in (7) can be interpreted as the loss-averse newsvendor’s marginal utility of overage and underage losses.

Uniform distribution: Suppose X is uniformly distributed between 0 and D . Then $F(x) = x/D$, and $F(ax) = aF(x)$. Therefore, $q_1(Q_1^*) = wQ_1^*$, $q_2(Q_1^*) = (1 + s - w)Q_1^*/s$, and $F(Q_1^*) = (1 + s - w)/(1 + s)$. Note that if $q_2(Q_1^*) \geq D$, then $\bar{F}(q_2(Q_1^*)) = 0$ and $g(s, w) < 0$.

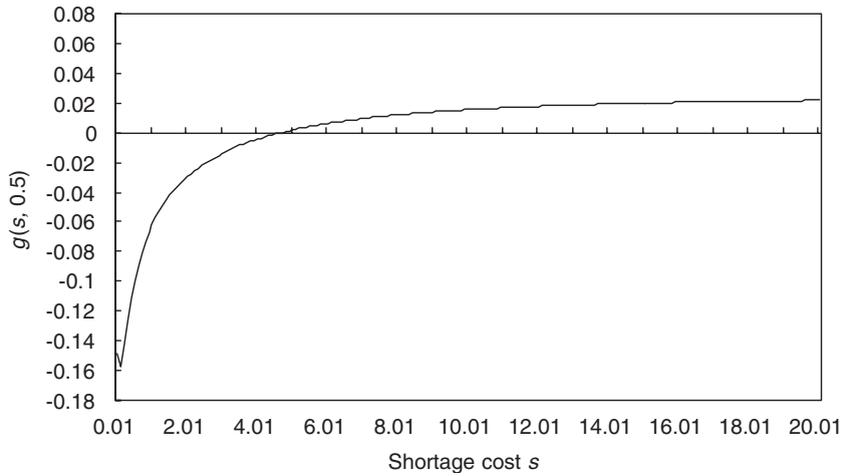


Fig. 2. A plot of $g(s, 0.5)$ with s ranging between 0.01 and 20.01 in increments of 0.5. Demand is exponentially distributed.

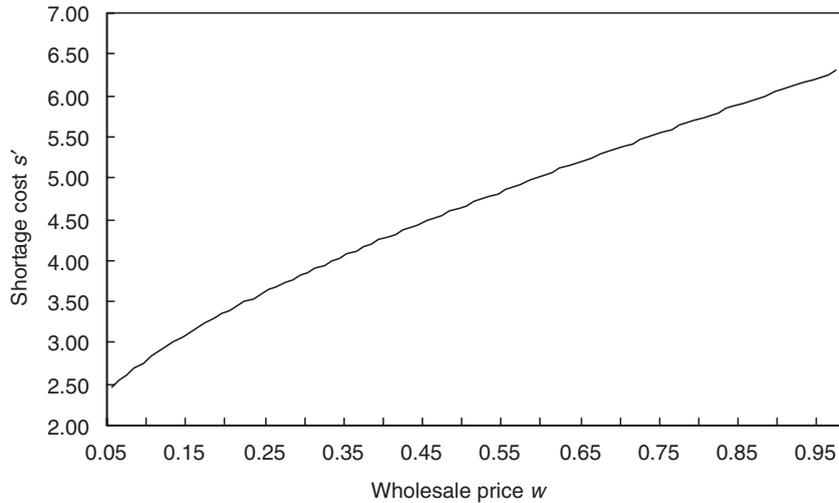


Fig. 3. A plot of s' as a function of wholesale price w ranging between 0.05 and 0.95 in increments of 0.05. Demand is exponentially distributed.

If $q_2(Q_1^*) < D$, then

$$\begin{aligned}
 g(s, w) &= (1 + s - w)\bar{F}(q_2(Q_1^*)) - wF(q_1(Q_1^*)) \\
 &= (1 + s - w) \left(1 - \left(\frac{1 + s - w}{s} \right) \right) \\
 &\quad \times \left(\frac{1 + s - w}{1 + s} \right) - w^2 \left(\frac{1 + s - w}{1 + s} \right) \\
 &= - \frac{(1 + s - w)(1 - w)^2}{s} < 0.
 \end{aligned}$$

In summary, $Q_\lambda^* < Q_1^*$ and $dQ_\lambda^*/d\lambda < 0$ for all s when X is uniformly distributed between 0 and D .

Exponential distribution: Suppose X is exponentially distributed with mean $1/\phi$. Then $f(x) = \phi e^{-\phi x}$, $\bar{F}(x) =$

$e^{-\phi x}$, and $\bar{F}(ax) = \bar{F}(x)^a$. Therefore, from $q_1(Q_1^*) = wQ_1^*$, $q_2(Q_1^*) = (1 + s - w)Q_1^*/s$, and $\bar{F}(Q_1^*) = w/(1 + s)$, it follows that

$$\begin{aligned}
 g(s, w) &= (1 + s - w)\bar{F}(q_2(Q_1^*)) - wF(q_1(Q_1^*)), \\
 &= (1 + s - w) \left(\frac{w}{1 + s} \right)^{(1 + s - w)/s} \\
 &\quad - w + w \left(\frac{w}{1 + s} \right)^w. \tag{8}
 \end{aligned}$$

The value of $g(s, w)$ is negative when s is close to zero and eventually becomes positive as s increases (see Fig. 2). We use numerical methods to obtain the positive value s' satisfying $g(s', w) = 0$ (see Fig. 3).

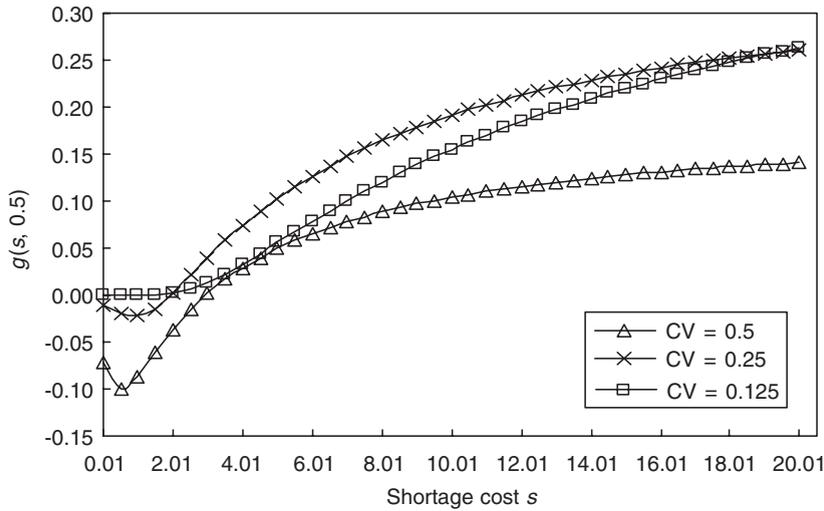


Fig. 4. A plot of $g(s, w)$ as a function of s with $w = 0.5$ and s ranging between 0.01 and 20.01 in increments of 0.5. Demand conforms to a truncated normal distribution with coefficient of variation $CV = 0.125, 0.25,$ and 0.5 .

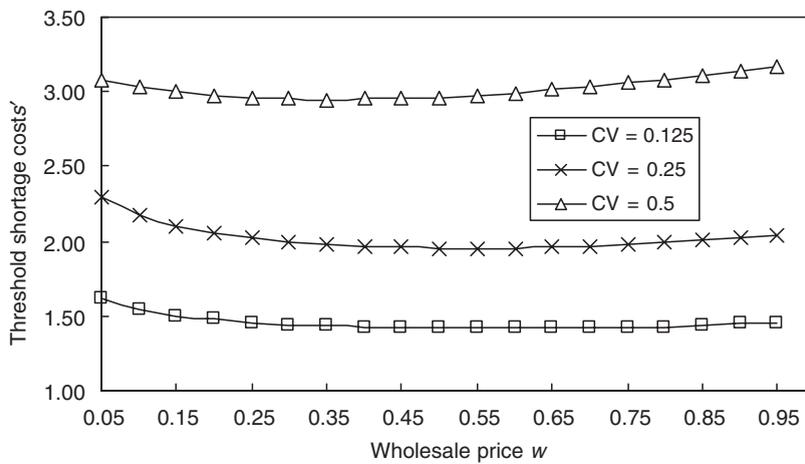


Fig. 5. A plot of s' as a function of wholesale price w ranging between 0.05 and 0.95 in increments of 0.05. Demand conforms to a truncated normal distribution with coefficient of variation $CV = 0.125, 0.25,$ and 0.5 .

While the complexity of (8) inhibits analytical conclusions, our numerical tests suggest that $g(s, w) > 0$ for $s > s'$ (and $g(s, w) < 0$ for $s < s'$), i.e., $Q_\lambda^* > Q_1^*$ and $dQ_\lambda^*/d\lambda > 0$ when $s > s'$.

Truncated normal distribution: We use similar numerical methods to find the threshold shortage cost s' for a truncated normal distribution, i.e., $X = \max\{Z, 0\}$ where Z is a normally distributed random variable with coefficient of variation ($CV = \sigma/\mu$) equal to 0.125, 0.25, and 0.5 (see Figs. 4 and 5). We note that for fixed coefficient of variation CV , the value of s' is unaffected by changes in the mean μ . This can be seen by letting

$z_1 = (Q_1^* - \mu)/\sigma$ and observing that $g(s', w) = (1 + s' - w)\bar{F}(q_2(Q_1^*)) - wF(q_1(Q_1^*)) = 0$ can be rewritten as

$$\begin{aligned} & (1 + s' - w)/w \\ &= \frac{F(wQ_1^*)}{\bar{F}\left(\frac{1 + s' - w}{s'}Q_1^*\right)} \\ &= \frac{F\left(\mu + \left[wz_1 - \frac{(1 - w)}{CV}\right]\sigma\right)}{\bar{F}\left(\mu + \left[\frac{1 + s' - w}{s'}z_1 + \frac{1 - w}{s'CV}\right]\sigma\right)}. \end{aligned} \tag{9}$$

The value of the right-hand side depends solely on the terms in brackets, and is unaffected by proportional increases or decreases in μ and σ (which leave CV unchanged). Furthermore, by substituting μCV for σ in the right-hand side and taking the derivative with respect to CV, it can be shown that s' is increasing (decreasing) in CV when $z_1 > 0$ ($z_1 < 0$). This result in combination with Fig. 5 implies that s' is increasing in demand uncertainty for $w \in \{0.05, 0.10, \dots, 0.95\}$.¹

Several general properties are evident in the above results. First, bias cannot be positive unless shortage cost is positive. Second, the direction of bias is independent of the sensitivity to losses as measured by λ and, given bias is present, the magnitude of bias is increasing in λ . Also, if the loss-averse newsvendor orders the same as the risk-neutral newsvendor, then the loss-averse optimal order quantity is unaffected by changes in λ . In essence, the presence of bias means that at Q_1^* , marginal underage and overage loss are not equal, and the term that dominates determines the direction of bias. One might suspect that loss aversion is associated with a smaller order quantity (relative to risk-neutral), or at least an order quantity that differs from risk-neutral. The results tell us that this suspicion does not always hold, and that bias depends both on the probability distribution of demand as well as the relative values of price and cost parameters.

For more detailed conclusions, we focus on the case of normally distributed demand. Due to the central limit theorem, normally distributed demand is arguably a reasonable approximation for a wide variety of settings in practice. We see that the likelihood of positive bias is increasing in shortage cost and decreasing in relative demand uncertainty. The impact of increasing s is not too surprising and is also evident in exponential demand; as s increases, the potential for loss due to insufficient stock eventually dominates the concern for overage loss (e.g., when $s \geq s'$). The role of uncertainty in the direction of bias is perhaps less obvious a priori. Clearly, uncertainty in demand contributes to costs, and reductions in demand uncertainty help improve newsvendor performance. It is interesting that reductions in demand uncertainty may also translate into a favorable effect for the supplier as bias shifts from negative to positive.

¹ Note that z_1 is increasing in s' . As illustrated in Fig. 5, at $\text{CV}=0.125$ and $w \in \{0.05, .010, \dots, 0.95\}$, s' is increasing in CV, so $z_1 > 0$ for $\text{CV} \geq 0.125$ and $w \in \{0.05, .010, \dots, 0.95\}$. If $z_1 < 0$ for some $\text{CV} < 0.125$ and $w \in \{0.05, .010, \dots, 0.95\}$, then s' decreases as CV increases to 0.125, leaving $z_1 < 0$, which is a contradiction.

On the flipside, the direction of bias does not change as the newsvendor's market increases or shrinks as long as the relative uncertainty in demand is stable. In addition, the numerical results suggest that the direction of bias is relatively insensitive to changes in wholesale price (e.g., observe the flat curves in Fig. 5). The results indicate that a supplier tactic predicated on the direction of bias may be fairly robust with respect to seasonal demand patterns and price changes.

Based upon these numerical results, we could classify products into two broad categories: the *low-shortage-cost* product and the *high-shortage-cost* product. We could list computers, food, and shirts as examples of low-shortage-cost products and airline meals, hotel rooms, and travel packages as *high-shortage-cost* products, e.g., in the airline industry, the cost per meal varies between \$2 and \$12 but the per meal shortage cost is around \$120 [32]. For high-shortage-cost products, a loss-averse newsvendor may order more than the risk-neutral newsvendor, whereas for low-shortage-cost products, loss aversion will cause the newsvendor to order less than a risk-neutral newsvendor.

4. Comparative statics

To gain more insights, in this section we investigate comparative statics of price changes on the loss-averse newsvendor's optimal order quantity.

Theorem 3. *The optimal order quantity Q_λ^* has the following relationships with price and cost parameters:*

- (i) Q_λ^* is increasing in s ;
- (ii) if $\bar{F}(Q_\lambda^*) + (\lambda - 1)(\bar{F}(q_2(Q_\lambda^*)) - q_2(Q_\lambda^*)f(q_2(Q_\lambda^*))) + (w/p)q_1(Q_\lambda^*)f(q_1(Q_\lambda^*))) < 0$, then $dQ_\lambda^*/dp < 0$, otherwise $dQ_\lambda^*/dp \geq 0$;
- (iii) if $(\lambda - 1)[\bar{F}(q_2(Q_\lambda^*)) - q_2(Q_\lambda^*)f(q_2(Q_\lambda^*))] + F(q_1(Q_\lambda^*)) + q_1(Q_\lambda^*)f(q_1(Q_\lambda^*)) + 1 < 0$, then $dQ_\lambda^*/dw < 0$, otherwise $dQ_\lambda^*/dw \geq 0$.

It is not surprising to see from Theorem 3(i) that the loss-averse newsvendor's optimal order quantity is increasing in shortage cost. This result also holds in the risk-neutral newsvendor model. However, Theorem 3(ii) and (iii) identify necessary and sufficient conditions under which the optimal order quantity decreases in retail price p and increases in wholesale price w . These results differ from the risk-neutral newsvendor model where the optimal order quantity is

always decreasing in wholesale price and increasing in retail price.

We next use a few numerical examples based upon uniform, normal, and exponential distributions to illustrate the loss-averse newsvendor's decision bias as retail price or wholesale price changes (see Figs. 6–8). For uniform distribution, we assume demand $X \in [0, D]$ with a mean $D/2 = 100$; for normal, we assume demand is a truncated normal random variable with a mean of $\mu = 100$ and a standard deviation of $\sigma = \{12.5, 25, 50\}$; and for exponential, we assume the mean demand $1/\phi = 100$. Since the newsvendor's optimal order quantity is qualitatively similar for various loss aversion levels $\lambda = \{2, 3, 4, 5\}$, we only report the results for $\lambda = 2$.

From Figs. 6–8, we find that the loss-averse newsvendor exhibits different ordering behavior from the risk-neutral newsvendor as retail price p or wholesale price w changes. More specifically, our numerical results show that the loss-averse retailer's optimal order quantity may decrease as p increases. This result contrasts with the risk-neutral newsvendor whose optimal order quantity always increases in p and suggests that this newsvendor decision bias towards changing retail price exists under the commonly used uniform, normal, and exponential distributions in practice. Our numerical results also show that the loss-averse retailer's optimal order quantity may increase as w increases. This result also contrasts with the risk-neutral newsvendor whose optimal order quantity always decreases in w .

To consider why the newsvendor's order quantity in Fig. 7 increases in $w \in [0.90, 0.95]$ when $s = 0.1$, suppose that wholesale price is currently 0.90 and the supplier increases the price to 0.95. The risk neutral newsvendor observes that the unit overage cost increases, unit underage cost decreases, and thus reduces his order quantity. Now imagine the perceived impact of this change by the loss-averse newsvendor given that the order quantity remains tentatively unchanged. The lower margin means that the probability of overage loss increases and, as noted earlier, unit overage cost increases and unit underage cost decreases. All these factors favor a reduction in the order quantity. However, the lower margin also causes the probability of underage loss to increase, and in this example, the increase is drastic relative to the increase in overage probability (i.e., from 7% to 45% versus 6–8%). This effect dominates the loss-averse newsvendor's perceptions and he increases his order quantity. Similar arguments explain why the order quantity decreases as retail price increases in Fig. 7. Of particular relevance to this paper is the

recent work of Buzacott et al. [33]. The authors show that given the mean–variance (MV) criterion, the optimal order quantity may decrease in selling price. Our result in Fig. 7 shows that the property observed by Buzacott et al. [33] for an MV model extends to the loss aversion framework.

The main lesson from the numerical examples in this and the previous section is that structural results from the risk-neutral newsvendor model do not necessarily extend to the loss-aversion, and that consideration of alternative choice models may help explain some decision biases in real-world newsvendor problems. For example, Fisher and Raman [3] observe that managers of a skiwear manufacturer, Sports Obermeyer, order systematically less than the risk-neutral manager's order quantity, a bias that may possibly be attributed to loss aversion. On the other hand, Patsuris [4] reports that despite the bad economy in 2001, many retail chains continue to add stores and order more unnecessary supply. Bear Stearns retail analyst Brian Tunick explains this phenomenon as the fact that managers have shareholders in mind: “Despite the downturn, they keep adding stores to get their shares awarded growth valuations by the market. When a company stops growing, [it gets] a multiple of 7 to 10 instead of something like 15 to 20” [4]. In this situation, managers of retailer stores may perceive a high shortage cost due to the negative impact on stock price if sales are stunted by insufficient supply. High shortage cost is conducive to positive bias and may help explain the excessive inventory investment.

We caution that loss aversion is one of multiple possible choice models that may help explain biases in human decision-making. More work on appropriate choice models in newsvendor settings is warranted. For example, Schweizer and Cachon [5] conduct a series of experiments that require subjects to solve newsvendor problems. They find that subjects, depending on the experimental setting, may consistently order more or less than the risk-neutral newsvendor. While this over and under ordering characteristic is also present in our results, the authors find that the biases in ordering decisions cannot be explained by loss aversion.

5. Conclusion

This paper investigates an alternative choice model (i.e., loss aversion) to describe newsvendor's decision-making behavior under demand uncertainty. We use a simple “kinked” piecewise-linear loss aversion utility function to study the single-period newsvendor model

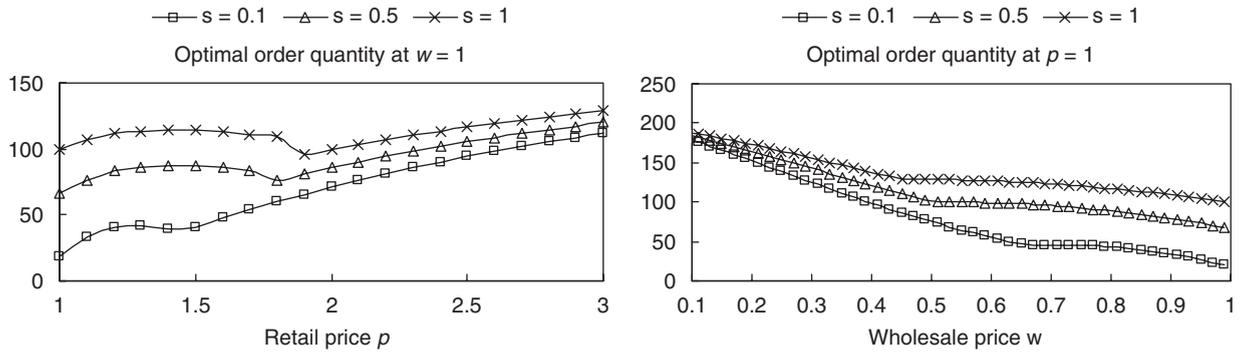


Fig. 6. A plot of newsvendor’s optimal order quantity as a function of retail price p ranging between 1 and 3 in increments of 0.1, and as a function of wholesale price w ranging between 0.01 and 1 in increments of 0.02. Demand conforms to a uniform distribution with $D = 100$. Loss aversion level $\lambda = 3$.

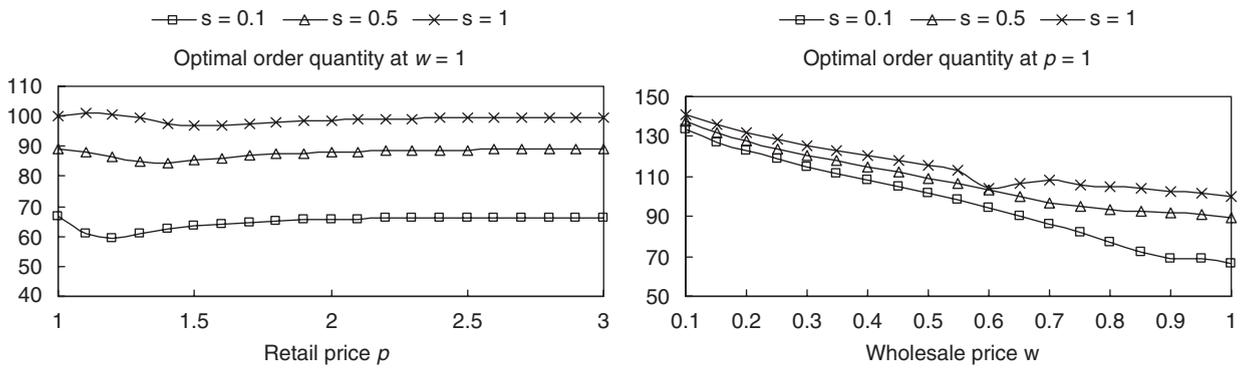


Fig. 7. A plot of newsvendor’s optimal order quantity as a function of retail price p ranging between 1 and 3 in increments of 0.1, and as a function of wholesale price w ranging between 0.1 and 1 in increments of 0.05. Demand conforms to a truncated normal distribution with mean 100 and standard deviation 25. Loss aversion level $\lambda = 3$.

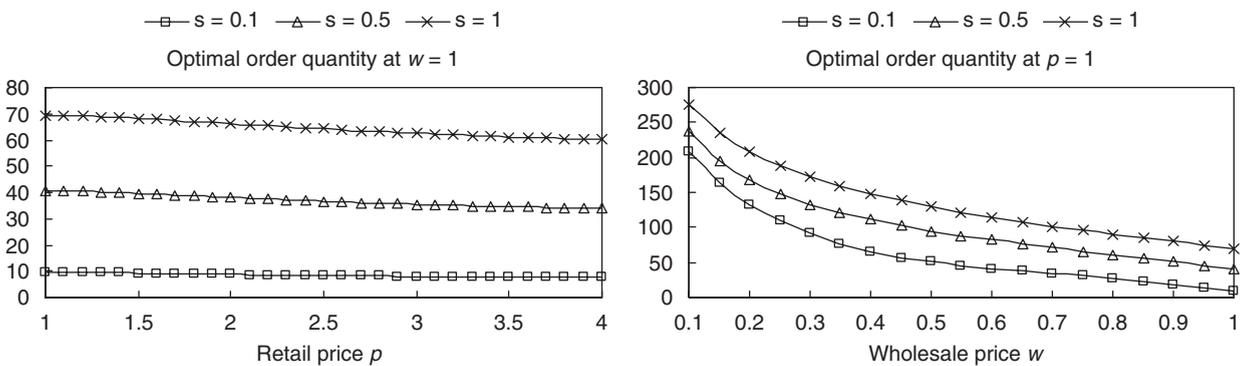


Fig. 8. A plot of newsvendor’s optimal order quantity as a function of retail price p ranging between 1 and 3 in increments of 0.1, and as a function of wholesale price w ranging between 0.1 and 1 in increments of 0.05. Demand conforms to an exponential distribution with mean 100. Loss aversion level $\lambda = 3$.

and show when, how, and why loss aversion causes the decision bias in the newsvendor problem. More specifically, we find (1) when shortage cost is low, a loss-

averse newsvendor will order less than a risk-neutral newsvendor and the more loss-averse, the less his optimal order quantity, and (2) when shortage cost is high,

a loss-averse newsvendor will order more than a risk-neutral newsvendor and the more loss-averse, the more his optimal order quantity. We also find that the loss-averse newsvendor’s optimal order quantity may increase in wholesale price and decrease in retail price, which can never occur in the risk-neutral newsvendor model. In essence, the presence of the decision bias in the loss-averse newsvendor problem can be explained by the fact that the loss-averse newsvendor is more sensitive to losses than gains and his marginal utility of underage and overage loss are not equal, and the term that dominates determines the direction of bias.

Future research on the appropriateness and effects of alternative choice models is warranted. In addition, future research should consider a supply chain comprised of manufacturer(s) selling to multiple competing loss-averse retailers. Such investigations may lead to new policies for improving supply chain efficiency. Finally, we wish to note that the loss-averse nature of the newsvendor might also be captured by making the unit shortage cost an increasing (convex) function of the shortage quantity and the unit salvage value a decreasing (concave) function of the unsold quantity. It would be interesting to see whether or not the newsvendor decision bias is qualitatively different from the loss aversion utility function employed in this paper.

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Appendix

Proof of Lemma 1. (i) For any $q < Q \in I$, from the payoff function (2), $\pi_-(q, Q) = q - wQ = 0$ iff $q = q_1(Q) = wQ < Q$. Hence, there always exists a breakeven quantity $q_1(Q) \in I$. Since $\pi_-(q, Q)$ is strictly increasing in q when $q < Q$, if $q_1(Q) < q < Q$, then $\pi_-(q, Q) > 0$ and if $q < q_1(Q)$, then $\pi_-(q, Q) < 0$. (ii) For any $q > Q$, from (2), $\pi_+(q, Q) = (1 + s - w)Q - sq = 0$ iff $q = q_2(Q) = (1 + s - w)Q/s$. If $s \geq (1 - w)Q/(\sup I - Q)$, then $q_2(Q) \leq \sup I$. Hence, there exists another breakeven quantity $q_2(Q) \in I$. Since $\pi_+(q, Q)$ is strictly decreasing in q when $q > Q$, if $Q < q < q_2(Q)$, then $\pi_+(q, Q) > 0$ and if $q > q_2(Q)$, then $\pi_+(q, Q) < 0$. Similarly, if $s < (1 - w)Q/(\sup I - Q)$, then $q_2(Q) > \sup I$. Hence,

the other breakeven quantity does not exist in I and $\pi_+(q, Q) > 0$ for all $q \in (Q, \sup I)$. \square

Proof of Theorem 1. (i) If $q_2(Q) \leq \sup I$, then after mapping the retailer’s payoff function (2) into the expected utility function (4), we can rewrite (4) as follows:

$$\begin{aligned}
 & E[U(\pi(X, Q))] \\
 &= \int_0^Q (x - wQ)f(x) dx \\
 &+ \int_Q^{\sup I} [(1 - w)Q - s(x - Q)]f(x) dx \\
 &+ (\lambda - 1) \left(\int_0^{q_1(Q)} (x - wQ)f(x) dx \right. \\
 &\left. + \int_{q_2(Q)}^{\sup I} [(1 - w)Q - s(x - Q)]f(x) dx \right). \tag{10}
 \end{aligned}$$

After taking the first derivative of (10) with respect to Q and applying the chain rule, we get:

$$\begin{aligned}
 & dE[U(\pi(X, Q))]/dQ \\
 &= -wF(Q) + (1 + s - w)\bar{F}(Q) + (\lambda - 1) \\
 &\times \left(-wF(q_1(Q)) + (q_1(Q) - wQ)f(q_1(Q)) \right. \\
 &\times \frac{dq_1(Q)}{dQ} + (1 + s - w)\bar{F}(q_2(Q)) - [(1 - w)Q \\
 &\left. - s(q_2(Q) - Q)]f(q_2(Q)) \frac{dq_2(Q)}{dQ} \right). \tag{11}
 \end{aligned}$$

Since from Lemma 1, $q_1(Q) = wQ$ and $q_2(Q) = (1 + s - w)Q/s$, we get

$$\begin{aligned}
 & (q_1(Q) - wQ)f(q_1(Q)) \frac{dq_1(Q)}{dQ} = 0 \quad \text{and} \\
 & [(1 - w)Q - s(q_2(Q) - Q)]f(q_2(Q)) \frac{dq_2(Q)}{dQ} = 0.
 \end{aligned}$$

Therefore, we can rewrite (11) as follows:

$$\begin{aligned}
 & dE[U(\pi(X, Q))]/dQ \\
 &= (1 + s - w)\bar{F}(Q) - wF(Q) + (\lambda - 1) \\
 &\times [(1 + s - w)\bar{F}(q_2(Q)) - wF(q_1(Q))]. \tag{12}
 \end{aligned}$$

After taking the second derivative of $E[U(\pi(X, Q))]$ with respect to Q , we get:

$$d^2E[U(\pi(X, Q))]/dQ^2 = -(1+s)f(Q) - (\lambda - 1) \left[w^2 f(q_1(Q)) + \frac{(1+s-w)^2 f(q_2(Q))}{s} \right]. \tag{13}$$

From (12) and (13), and noting $s > w - 1$,² we see that $dE[U(\pi(X, 0))]/dQ > 0$, $dE[U(\pi(X, \sup I))]/dQ < 0$, and $d^2E[U(\pi(X, Q))]/dQ^2 < 0$ for all $Q \in I$. Hence, if $q_2(Q) \leq \sup I$, then there exists a unique optimal order quantity Q_λ^* that satisfies the first-order condition (5).

(ii) If $q_2(Q) > \sup I$, then $\int_{q_2(Q)}^{\sup I} [(1-w)Q - s(x-Q)]f(x) dx = 0$. Hence, we can rewrite (4) as follows:

$$E[U(\pi(X, Q))] = \int_0^Q (x-wQ)f(x) dx + \int_Q^{\sup I} [(1-w)Q - s(x-Q)]f(x) dx + (\lambda - 1) \int_0^{q_1(Q)} (x-wQ)f(x) dx. \tag{14}$$

Similarly, after taking the first derivative of (14) with respect to Q and applying the chain rule, we get:

$$dE[U(\pi(X, Q))]/dQ = (1+s-w)\bar{F}(Q) - wF(Q) - (\lambda-1)wF(q_1(Q)). \tag{15}$$

After taking the second derivative of $E[U(\pi(X, Q))]$ with respect to Q , we get:

$$d^2E[U(\pi(X, Q))]/dQ^2 = -(1+s)f(Q) - (\lambda - 1)w^2 f(q_1(Q)). \tag{16}$$

From (15) and (16), and noting $s > w - 1$, we see that $dE[U(\pi(X, 0))]/dQ > 0$, $dE[U(\pi(X, \sup I))]/dQ < 0$, and $d^2E[U(\pi(X, Q))]/dQ^2 < 0$ for all $Q \in I$. Since $\bar{F}(q_2(Q)) = 0$ if $q_2(Q) > \sup I$, we can rewrite (15) as

follows:

$$dE[U(\pi(X, Q))]/dQ = (1+s-w)\bar{F}(Q) - wF(Q) + (\lambda - 1) \times [(1+s-w)\bar{F}(q_2(Q)) - wF(q_1(Q))],$$

which is the same as (12). Hence, if $q_2(Q) > \sup I$, then there exists a unique optimal order quantity Q_λ^* that satisfies the first-order condition (5). \square

Proof of Theorem 2. After substituting Q_1^* into $dE[U(\pi(X, Q))]/dQ$, we get

$$dE[U(\pi(X, Q_1^*))]/dQ = (1+s-w)\bar{F}(Q_1^*) - wF(Q_1^*) + (\lambda - 1) \times [(1+s-w)\bar{F}(q_2(Q_1^*)) - wF(q_1(Q_1^*))].$$

From (6), $dE[U(\pi(X, Q_1^*))]/dQ$ reduces to

$$dE[U(\pi(X, Q_1^*))]/dQ = (\lambda - 1)[(1+s-w)\bar{F}(q_2(Q_1^*)) - wF(q_1(Q_1^*))]. \tag{17}$$

Therefore, if $(1+s-w)\bar{F}(q_2(Q_1^*)) > wF(q_1(Q_1^*))$, then $dE[U(\pi(X, Q_1^*))]/dQ > 0$. Since $E[U(\pi(X, Q))]$ is concave, $Q_1^* < Q_\lambda^*$, which in turn implies $(1+s-w)\bar{F}(Q_\lambda^*) - wF(Q_\lambda^*) < 0$. Thus, from the first-order condition (5), $(1+s-w)\bar{F}(q_2(Q_\lambda^*)) - wF(q_1(Q_\lambda^*)) > 0$. Therefore, by the implicit function theorem,

$$dQ_\lambda^*/d\lambda = \frac{d^2E[U(\pi(X, Q_\lambda^*))]/dQ d\lambda}{-d^2E[U(\pi(X, Q_\lambda^*))]/dQ^2} = \frac{(1+s-w)\bar{F}(q_2(Q_\lambda^*)) - wF(q_1(Q_\lambda^*))}{-d^2E[U(\pi(X, Q_\lambda^*))]/dQ^2} > 0. \tag{18}$$

Similarly, we can prove that if $(1+s-w)\bar{F}(q_2(Q_1^*)) \leq wF(q_1(Q_1^*))$, then $dQ_\lambda^*/d\lambda \leq 0$. \square

Proof of Corollary 1. If $s \leq 0$, then $(1+s-w)\bar{F}(q_2(Q_1^*)) = 0 < wF(q_1(Q_1^*))$. From Theorem 2, $Q_\lambda^* < Q_1^*$ and $dQ_\lambda^*/d\lambda < 0$ if and only if $(1+s-w)\bar{F}(q_2(Q_1^*)) < wF(q_1(Q_1^*))$. Since $\lim_{s \rightarrow 0} (1+s-w)\bar{F}(q_2(Q_1^*)) = 0$, then for any $0 < \varepsilon < wF(q_1(Q_1^*))$, there exists $s' > 0$ such that $(1+s-w)\bar{F}(q_2(Q_1^*)) < \varepsilon < wF(q_1(Q_1^*))$ for all $s < s'$. \square

² Note that if $s \leq w - 1$, then the newsvendor can make as much or more money on lost sales, and the decision problem is trivial, i.e., order nothing until after demand is realized and receive payment $-s \geq 1 - w$ on each unsold unit.

Proof of Theorem 3. (i) By the implicit function theorem, from (5), we get:

$$\begin{aligned} \frac{dQ_\lambda^*}{ds} &= \frac{d^2 E[U(\pi(X, Q_\lambda^*))]/dQ ds}{-d^2 E[U(\pi(X, Q_\lambda^*))]/dQ^2} \\ &= \frac{\bar{F}(Q_\lambda^*) + (\lambda - 1) \left[\bar{F}(q_2(Q_\lambda^*)) + \frac{(p-w)q_2(Q_\lambda^*)}{s} f(q_2(Q_\lambda^*)) \right]}{-d^2 E[U(\pi(X, Q_\lambda^*))]/dQ^2} \\ &> 0. \end{aligned}$$

(ii)–(iii) Similarly, we get:

$$dQ_\lambda^*/dp = \frac{\bar{F}(Q_\lambda^*) + (\lambda - 1) \left[\bar{F}(q_2(Q_\lambda^*)) - q_2(Q_\lambda^*) f(q_2(Q_\lambda^*)) + \left(\frac{w-v}{p-v} \right) q_1(Q_\lambda^*) f(q_1(Q_\lambda^*)) \right]}{-d^2 E[U(\pi(X, Q_\lambda^*))]/dQ^2}$$

and

$$dQ_\lambda^*/dw = \frac{-1 - (\lambda - 1) [\bar{F}(q_2(Q_\lambda^*)) - q_2(Q_\lambda^*) f(q_2(Q_\lambda^*)) + F(q_1(Q_\lambda^*)) + q_1(Q_\lambda^*) f(q_1(Q_\lambda^*))]}{-d^2 E[U(\pi(X, Q_\lambda^*))]/dQ^2}.$$

So if $\bar{F}(Q_\lambda^*) + (\lambda - 1) [\bar{F}(q_2(Q_\lambda^*)) - q_2(Q_\lambda^*) f(q_2(Q_\lambda^*)) + ((w - v)/(p - v)) q_1(Q_\lambda^*) f(q_1(Q_\lambda^*))] < 0$, then we have $dQ_\lambda^*/dp > 0$, otherwise $dQ_\lambda^*/dp \leq 0$. Similarly, if $(\lambda - 1) [\bar{F}(q_2(Q_\lambda^*)) - q_2(Q_\lambda^*) f(q_2(Q_\lambda^*)) + F(q_1(Q_\lambda^*)) + q_1(Q_\lambda^*) f(q_1(Q_\lambda^*))] + 1 < 0$, then $dQ_\lambda^*/dw < 0$, otherwise $dQ_\lambda^*/dw \geq 0$. \square

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