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Markdown money contracts for perishable goods with clearance pricing

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ABSTRACT

It is common in practice that retailers liquidate unsold perishable goods via clearance pricing. Markdown money is frequently used between manufacturers and retailers in such a supply chain setting. It is a form of rebate from a manufacturer to subsidize a retailer's clearance pricing after the regular season. Two forms of markdown money are *percent markdown money*, in which the markdown money is limited to only a certain percentage of the retail price markdown, and *quantity markdown money*, which is essentially a *buyback contract* or *returns policy* with a rebate credit paid to the retailer for each unsold unit after the regular season. We show both forms of markdown money contracts can coordinate the supply chain and we discuss their strengths and limitations.

1. Introduction

The volatile market of perishable goods (e.g., fashion apparel, consumer electronics, personal computers, toys, books, and CDs) is featured by uncertain demand, long lead time, and a short selling season. The retailer initially purchases products from the manufacturer before the selling season begins and has a great chance of facing overstock. Since perishable goods will lose value in the eyes of the customer after the regular season, it is common in practice that retailers liquidate excess inventory via clearance pricing (Forest et al., 2003; Kratz, 2005; Rozhon, 2005).

Clearance pricing is an important price promotion tool for the retailer to enhance sales of unsold units after the regular selling season. Since the retailer makes less money from clearance sales than the regular season, it is common in practice that large retailers (e.g., May, Federated, Kohl's, Saks, and J.C. Penney) demand a rebate called *markdown money* (or markdown allowance) from the manufacturer to subsidize their clearance sales (Kratz, 2005; Rozhon, 2005). Markdown money is prevalent in industries selling perishable goods, e.g., fashion apparel, cosmetics and fragrances, toys, specialty productions, and over-the-counter medications (Tsay, 2001).

One form of markdown money is *quantity markdown money* (QMM), in which the manufacturer pays a rebate credit to the retailer for each unsold unit at the end of the regular selling season. QMM contracts are also known as buyback contracts or returns policies in the literature when the retailer, not the manufacturer, salvages overstock at the end of the regular selling season (Cachon, 2003, p. 242).

Another form of markdown money called *percent markdown money* (PMM), in which the markdown money paid to the retailer is a certain percentage of the retail price markdown, i.e., the difference between the regular selling price and clearance price. For example, Rozhon (2005) reports that in the fashion industry, a strong retailer may demand as much as 100% of their retail price markdowns from a supplier. The main difference between a PMM and a QMM contract is that the rebate depends on the end of season clearance price in a PMM contract whereas the rebate is specified at the start of the season in a QMM (or buyback) contract. If the end of season clearance price is known at the start of the season when the contract terms are set, then PMM and QMM contracts are identical.

The main purpose of this paper is to address the following issues between the manufacturer and the retailer regarding the markdown money contract in practice: (1) the impact of the magnitude of the markdown money and (2) the relative performance of PMM and QMM contracts. Regarding the first issue, if the manufacturer's markdown money is too little, then the retailer will order fewer products than the manufacturer would like; however, if the retailer demands too much markdown money, then the manufacturer's performance may be hurt because of the large payment to the retailer. For example, an executive for one of the best-known apparel makers reports that if the manufacturer refuses the markdown money proposed by the retailers, then they will order 5% percent less than usual. On the other hand, two major clothing companies, Kellwood and Jones, have warned of lower earnings in part because of post-holiday markdown money payments to

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the retailer (Rozhon, 2005). Thus, it is important to know whether or not the QMM and PMM contracts can improve supply chain performance and allow both parties to benefit. Regarding the second issue, there is a vast literature on the QMM contracts (Cachon, 2003; Lariviere, 1999), but this work assumes that the end of season salvage value (e.g., clearance price) is exogenous. In practice, clearance price is commonly affected by market conditions observed during the selling season, in which case PMM and QMM contracts have different impacts on decisions and expected profits. We seek to understand the merits of each type of contract when clearance price is endogenous.

Cachon and Kök (2007) propose and analyze a clearance pricing model. Their work lends insight into how a retailer can apply the powerful framework of the newsvendor model, which relies on an exogenous salvage value assumption, in settings where salvage value depends on the retailer's end-of-season clearance price decision. This paper builds on the clearance pricing model of Cachon and Kök (2007), but rather than addressing the question of how to estimate salvage value when clearance price is endogenous, we investigate the relative performance of two types of markdown money contracts. In summary, the previous research on perishable goods treats markdown money and clearance pricing separately. This paper draws on both these literatures to evaluate the relative performance of markdown money contracts.

This paper is organized as follows. In Section 2, we briefly review the relevant literature. In Section 3, we present our model preliminaries and analyze supply chain models under a general clearance demand function. To identify additional insights, in Section 4, we analyze supply chain models under a special linear clearance demand function. Finally, in Section 5, we summarize our results and identify research opportunities for future study.

2. Literature review

The literature related to this paper can be divided into two categories: papers on the newsvendor problem and its extensions and papers on supply chain contracts.

The single-period newsvendor model has been well studied in the literature (see Porteus, 1990; Lee and Nahmias, 1993; Khouja, 1999; Petruzzi and Dada, 1999 for more detailed reviews). In those models, it is commonly assumed that the salvage value for excess inventory is fixed and exogenously given. Recently, a number of attempts have been made to extend the understanding of the newsvendor model. The paper most closely related to our research is Cachon and Kök (2007), who study a newsvendor model with clearance pricing for the leftover inventory at the end of the selling season. In contrast with the traditional newsvendor model, they treat the salvage value (or clearance price) as a decision variable and focus on methods for estimating salvage value in the newsvendor problem.

Supply chain contract models have received much attention from researchers recently. In this section, we only focus on returns policies and markdown money, which are most closely related to this research. We refer to Cachon (2003) for reviews of other types of supply chain contracts.

Returns policies, also known as buyback contracts, allow the retailer to return a certain amount of unsold goods to the manufacturer at the end of the selling season for a partial rebate credit. Returns policies are common in the distribution of perishable commodities with uncertain demand, such as books, magazines, newspapers, recorded music, computer hardware and software, greeting cards, and pharmaceuticals (Padmanabhan and Png, 1995). Pasternack (1985) is the first to study a returns policy. He shows that both full returns with full rebate credit and no returns are system suboptimal. The supply chain can be coordinated by an intermediate returns policy, e.g., partial returns with full rebate credit. Kandel (1996) studies two extreme contract schemes for allocation of responsibility for unsold inventory in a supply chain: the consignment contract and the no-return contract. He also shows if demand is stochastic and price-sensitive, then the supply chain cannot be fully coordinated by returns policies without retail price maintenance (i.e., allowing the manufacturer to dictate the retail price). Emmons and Gilbert (1998) study a price-sensitive multiplicative model of demand uncertainty for catalog goods and demonstrate that uncertainty tends to increase the retail price. They also show that under certain conditions, a manufacturer can increase her profit by offering a returns policy. Webster and Weng (2000) take the viewpoint of a manufacturer selling a short life-cycle product to a single retailer and describe risk free returns policies that, when compared to no returns, the retailer's expected profit is increased and the manufacturer's realized profit is ensured to be at least as large as when no returns are allowed. Donohue (2000) studies returns policies in a supply chain model with multiple production opportunities and improving demand forecasts. Lee et al. (2000) study dynamic optimal price protection policies for products subject to price reductions due to obsolescence (e.g., personal computers), which closely resemble returns policies. Taylor (2002) incorporates a buyback contract with a target sales rebate contract to coordinate the supply chain when demand is sensitive to the retailer sales effort. Krishnan et al. (2004) study a decentralized supply chain with retailer promotional effort. They show that returns policies alone will reduce the retailer's promotional incentives and adversely affect supply chain profits. However, under certain conditions, coupling returns policies with other channel mechanisms (e.g., promotional cost sharing) can coordinate the supply chain. Su and Zhang (2005) study a decentralized supply chain with strategic customers, who anticipate the seller's future clearance sales at a fixed salvage price and choose the best purchasing time to maximize their expected surplus. They show how contractual arrangements can be used to improve supply chain performance. Finally, Wang and Webster (2007) study a decentralized supply chain in which a single risk-neutral manufacturer is selling a perishable product to a single loss-averse retailer. They investigate a returns policy with a gain/loss sharing provision to coordinate the supply chain.

As pointed out by Cachon (2003), the name of returns policy or buyback contract is somewhat misleading since it implies physical returns of overstock at the end of the selling season, which only happens when the manufacturer's salvage value is higher than the retailer's. If the retailer's salvage value is higher than the manufacturer, then the retailer liquidates overstock and the manufacturer credits the retailer for those units, which is often referred to as *markdown money*contract (Tsay, 2001). To our knowledge, past research on buyback and markdown money contracts assumes that salvage value (e.g., clearance price) is exogenous. However, there are many settings where a retailer's salvage value is not fixed, but is based on a clearance price that is influenced by the observed demand during the selling season. In this paper we use the clearance pricing model of Cachon and Kök (2007). We extend the supply chain contract literature by allowing for endogenous salvage, and we find that the way in which markdown money is calculated (e.g., PMM versus QMM) leads to meaningful differences in performance.

3. Supply chain models with a general clearance demand function

In Section 3.1, we begin our analysis by investigating a vertically integrated firm that owns both manufacturer and retailer and acts as a central planner for the supply chain. This centralized control setting provides us a first-best solution that maximizes total supply chain profit. We then investigate a decentralized supply chain under a wholesale price-only contract. In Sections 3.2 and 3.3, we study the decentralized supply chain with the PMM contract and QMM contract, respectively.

3.1. Integrated and decentralized supply chains models with a wholesale price-only contract

Our integrated supply chain model builds on the model of Cachon and Kök (2007). We briefly introduce our model notation and assumptions, while referring interested readers to Cachon and Kök (2007) for model details.

We consider an integrated firm selling a perishable product with a selling season divided into two periods, a regular season T_1 and a clearance period T_2. At the beginning of T_1, the integrated firm produces q units of a single item at a quantity independent unit production and delivery cost c and sells at a unit retail price $p_1 > c$ in T_1. Like most of the supply chain contract models we assume the retail price p_1 in T_1 is fixed (e.g., Pasternack, 1985; Kandel, 1996; Lariviere, 1999; Lee et al., 2000; Tsay, 2001; Taylor, 2002; and Cachon and Kök, 2007). This assumption is practically reasonable if the retail market is highly competitive (e.g., the retailer acts as a price taker).

Let $\xi \in [0, \infty)$ be the realized demand in T_1. Let $F(\xi)$ be the strictly increasing and differentiable distribution function of demand and let $f(\xi)$ be the density function. If realized demand ξ is higher than the order quantity q, then all sales are lost without additional penalty; if realized demand ξ is lower than q, then the leftover inventory $I(q,\xi) = (q-\xi)^+$ will be carried over to the clearance period T_2. We let $I(q) = \int_0^q (q-\xi)\,\mathrm{d}F(\xi)$ be the expected leftover inventory.

Clearance demand $D_2(p_2,\xi)$ in T_2 is a deterministic function of the clearance price p_2 and the realized demand ξ in T_1. $D_2(p_2,\xi)$ is non-negative, differentiable, and decreasing in p_2. Hence, the inverse demand function exists, $p_2(s_2,\xi)$, where s_2 is actual sales in T_2. We assume revenue in T_2, $s_2 p_2(s_2,\xi)$, is concave in s_2 for all ξ; and $\hat{p}_2(\xi) < p_1$ for all ξ, where $\hat{p}_2(\xi) = \arg\max_{p_2}(p_2 D_2(p_2,\xi))$. We assume $D_2(p_2,\xi)$ is monotone in ξ for all p_2.

Let $\widehat{R}_2(s_2,\xi) = s_2 p_2(s_2,\xi)$ be the unconstrained revenue function. Let $\hat{s}_2(\xi)$ be the unconstrained optimal sales in T_2

$$\hat{s}_2(\xi) = \underset{s_2}{\operatorname{argmax}}\, \widehat{R}_2(s_2,\xi).$$

Then the integrated firm's revenue function, $R_2(q,\xi)$, given realized demand ξ in T_1, can be expressed as follows:

$$R_2(q,\xi) = \begin{cases} \hat{s}_2(\xi)p_2(\hat{s}_2(\xi),\xi), & 0 \leqslant \xi \leqslant \hat{\xi}(q), \\ I(q,\xi)p_2(I(q,\xi),\xi), & \hat{\xi}(q) < \xi \leqslant \tilde{\xi}(q), \\ I(q,\xi)p_1, & \tilde{\xi}(q) < \xi \leqslant q, \\ 0, & q < \xi, \end{cases}$$

where $\hat{\xi}(q)$ and $\tilde{\xi}(q)$ are defined in Cachon and Kök (2007). More specifically, if realized demand is lower than $\hat{\xi}(q)$, then the firm will only liquidate some leftover inventory at a clearance price $p_2 < p_1$ and hold back (destroy) the rest; if realized demand is higher than $\hat{\xi}(q)$ but lower than $\tilde{\xi}(q)$, then the firm will liquidate all leftover inventory; if realized demand is higher than $\tilde{\xi}(q)$, then the firm will liquidate all leftover inventory at the regular season retail price p_1; finally, if realized demand is higher than q, then there is no clearance sales.

Then, we can express the integrated firm's expected profit as follows:

$$\Pi(q) = -cq + R_1(q) + R_2(q), \tag{1}$$

where

$$R_1(q) = p_1(q - I(q)) \tag{2}$$

is expected revenue in T_1 with $I(q)$ being expected left over inventory, and

$$R_2(q) = \int_0^{\hat{\xi}(q)} \hat{s}_2(\xi)\hat{p}_2(\xi)\mathrm{d}F(\xi) + \int_{\hat{\xi}(q)}^{\tilde{\xi}(q)} p_2(I(q,\xi),\xi)I(q,\xi)\mathrm{d}F(\xi) + \int_{\tilde{\xi}(q)}^{q} p_1 I(q,\xi)\mathrm{d}F(\xi)$$

is expected revenue in T_2. Then it follows from Cachon and Kök (2007) that the optimal order quantity q^0 of the integrated firm is unique and satisfies

$$p_1 - c - p_1F(q^0) + \int_{\hat{\xi}(q^0)}^{\tilde{\xi}(q^0)} \frac{\partial}{\partial q}(I(q^0,\xi)p_2(I(q^0,\xi),\xi)\mathrm{d}F(\xi) + \int_{\tilde{\xi}(q^0)}^{q^0} p_1\,\mathrm{d}F(\xi) = 0. \tag{3}$$

Replacing the production cost c with the wholesale price w in (1), we have the following proposition on the retailer's optimal order quantity under the wholesale price-only contract.

Proposition 1. *The retailer's optimal order quantity q^* under the wholesale price-only contract is unique and satisfies*

$$p_1 - w - p_1F(q^*) + \int_{\hat{\xi}(q^r)}^{\tilde{\xi}(q^*)} \frac{\partial}{\partial q}(I(q^*,\xi)p_2(I(q^*,\xi),\xi)\mathrm{d}F(\xi) + \int_{\tilde{\xi}(q^*)}^{q^*} p_1\,\mathrm{d}F(\xi) = 0. \tag{4}$$

Furthermore, the retailer's optimal order quantity is smaller than the integrated firm's optimal stocking level, i.e., $q^* < q^0$.

Proof. The proof is straightforward so we omit it. □

The quantity distortion implied in Proposition 1 can be explained by *double marginalization* (Spengler, 1950), i.e., if the manufacturer sells at a wholesale price which is higher than her production cost, then the retailer will order less inventory than the optimal stocking level of the integrated supply chain and the total expected profit of the decentralized supply chain is lower than the integrated channel.

3.2. Decentralized supply chain models with a PMM contract

As shown in Section 3.1, since the wholesale price-only contract is system suboptimal, we next investigate the role of the PMM contract on the supply chain coordination. The PMM contract specifies that the manufacturer charges the retailer a unit wholesale price w and pays the retailer some markdown money M which is a percentage $\gamma \in (0,1)$ of the difference between the retailer's regular selling price p_1 and the clearance price p_2, or a percentage of the retailer's regular selling price p_1 for the inventory held back by the retailer with zero value. The timing of the supply chain events with the PMM contract are as follows:

1. Prior to the selling season, the manufacturer offers the retailer (possibly after negotiation) a PMM contract (w,γ).
2. The retailer places an order with the manufacturer at unit wholesale price w.
3. Production takes place and all finished goods are delivered to the retailer before the regular season T_1 begins.
4. The retailer sells the product at an exogenous retail price p_1 during the selling season. Regular season demand is realized and all unsold units are carried over to the clearance period T_2.
5. The retailer chooses a clearance price p_2 for clearance sales in T_2.

6. At the end of T_2, the retailer salvages the inventory he holds back and the manufacturer pays the retailer a percentage γ of revenues lost due to markdowns during T_2 and due to leftover inventory that is salvaged at the end of T_2.

The retailer's clearance revenue function is slightly different from the integrated firm because of the introduction of the PMM contract, under which the retailer is able to get additional compensation from the manufacturer for his clearance sales and inventory holdbacks. Similarly, we can express the retailer's revenue function, $R_2(q,\gamma,\xi)$, given realized demand ξ in T_1, as follows:

$$R_2(q,\gamma,\xi)=\begin{cases}(1-\gamma)\hat{s}_2(\xi)p_2(\hat{s}_2(\xi),\xi)+\gamma p_1 I(q,\xi), & 0\leqslant\xi\leqslant\hat{\xi}(q),\\ (1-\gamma)I(q,\xi)p_2(I(q,\xi),\xi)+\gamma p_1 I(q,\xi), & \hat{\xi}(q)<\xi\leqslant\tilde{\xi}(q),\\ I(q,\xi)p_1, & \tilde{\xi}(q)<\xi\leqslant q,\\ 0, & q<\xi.\end{cases}$$

Then, we can express the retailer's expected profit under the PMM contract as follows:

$$\Pi_r(q,w,\gamma)=-wq+R_1(q)+R_2(q,\gamma), \tag{5}$$

where $R_1(q)$ is define in (2) and

$$R_2(q,\gamma)=(1-\gamma)\int_0^{\hat{\xi}(q)}\hat{s}_2(\xi)p_2(\hat{s}_2(\xi),\xi)\mathrm{d}F(\xi)+(1-\gamma)\times\int_{\hat{\xi}(q)}^{\tilde{\xi}(q)}I(q,\xi)p_2(I(q,\xi),\xi)\mathrm{d}F(\xi)+\int_0^{\tilde{\xi}(q)}\gamma p_1 I(q,\xi)\mathrm{d}F(\xi)+\int_{\tilde{\xi}(q)}^{q}p_1 I(q,\xi)\mathrm{d}F(\xi).$$

Proposition 2. *The retailer's optimal order quantity q^γ under the PMM contract is unique and satisfies the following first-order condition:*

$$p_1-w-(1-\gamma)\left(p_1F(q^\gamma)-\int_{\hat{\xi}(q^\gamma)}^{\tilde{\xi}(q^\gamma)}\frac{\partial}{\partial q}(I(q^\gamma,\xi)p_2(I(q^\gamma,\xi),\xi))\mathrm{d}F(\xi)-\int_{\tilde{\xi}(q^\gamma)}^{q^\gamma}p_1\,\mathrm{d}F(\xi)\right)=0. \tag{6}$$

Furthermore, q^γ is increasing in γ.

Proof. After taking the first and second derivatives of $\Pi_r(q,w,\gamma)$ expressed in (5) with respect to q, we get

$$\mathrm{d}\Pi_r(q,w,\gamma)/\mathrm{d}q=p_1-w-(1-\gamma)\left(p_1F(q)-\int_{\hat{\xi}(q)}^{\tilde{\xi}(q)}\frac{\partial}{\partial q}(I(q,\xi)p_2(I(q,\xi),\xi))\mathrm{d}F(\xi)-\int_{\tilde{\xi}(q)}^{q}p_1\,\mathrm{d}F(\xi)\right)$$

and

$$\mathrm{d}^2\Pi_r(q,w,\gamma)/\mathrm{d}q^2=(1-\gamma)\int_{\hat{\xi}(q)}^{\tilde{\xi}(q)}\frac{\partial^2}{\partial q^2}(I(q,\xi)p_2(I(q,\xi),\xi))\mathrm{d}F(\xi)-(1-\gamma)\left(p_1-\frac{\partial}{\partial q}(I(q,\xi)p_2(I(q,\xi),\xi))|_{\xi=\tilde{\xi}(q)}\right)f(\tilde{\xi}(q))\tilde{\xi}'(q)-(1-\gamma)\frac{\partial}{\partial q}(I(q,\xi)p_2(I(q,\xi),\xi)|_{\xi=\hat{\xi}(q)}f(\hat{\xi}(q))\hat{\xi}'(q)<0.$$

Thus, $\Pi_r(q,w,\gamma)$ is concave in q, which implies there must exist a unique optimal retailer order quantity q^γ that satisfies the first-order condition, $\mathrm{d}\Pi_\gamma(q^\gamma,w,\gamma)/\mathrm{d}q=0$, i.e., Eq. (6).

Finally, by the Implicit Function Theorem, we have

$$\mathrm{d}q^\gamma/\mathrm{d}\gamma=\frac{\mathrm{d}^2\Pi_r(q^\gamma,w,\gamma)/\mathrm{d}q\,\mathrm{d}\gamma}{-\mathrm{d}^2\Pi_r(q^\gamma,w,\gamma)/\mathrm{d}q^2}=\frac{p_1F(\hat{\xi}(q^\gamma))+\int_{\hat{\xi}(q^\gamma)}^{\tilde{\xi}(q^\gamma)}\left(p_1-\frac{\partial}{\partial q}(I(q^\gamma,\xi)p_2(I(q^\gamma,\xi),\xi))\right)\mathrm{d}F(\xi)}{-\mathrm{d}^2\Pi_r(q^\gamma,w,\gamma)/\mathrm{d}q^2}>0.\quad\square$$

Proposition 2 shows that under the PMM contract, the retailer's total expected profit function is concave and there exists a unique optimal order quantity that maximizes the retailer's expected profit. In addition, the higher the manufacturer's markdown money percentage, the more the retailer would like to order from the manufacturer.

Proposition 3. *Consider the set of PMM contracts with*

$$\gamma_w=\frac{w-c}{p_1-c}. \tag{7}$$

(i) *The retailer orders the integrated supply chain optimal order quantity q^0, i.e., those contracts can coordinate the supply chain.*
(ii) *The manufacturer's expected profit is $\Pi_m(q^0,w,\gamma_w)=\gamma_w\Pi(q^0)$ and the retailer's expected profit is $\Pi_r(q^0,w,\gamma_w)=(1-\gamma_w)\Pi(q^0)$.*
(iii) *The set of PMM contracts (w,γ_w) can arbitrarily allocate integrated supply chain profit between the manufacturer, who receives share γ_w, and the retailer, who receives share $1-\gamma_w$.*
(iv) *γ_w is increasing in w and decreasing in both p_1 and c.*
(v) *For any clearance price $p_2\leqslant p_1$, the markdown money $M^*=\gamma_w(p_1-p_2)$ is less than the wholesale price w.*

Proof. (i) Let (w,γ_w) be the channel coordinating PMM contract under which the retailer's optimal order quantity q^γ is the same as the integrated supply chain's optimal order quantity q^0, i.e., $q^\gamma=q^0$. Then q^γ must satisfy $\mathrm{d}\Pi_r(q^\gamma,w,\gamma_w)/\mathrm{d}q=\mathrm{d}\Pi(q^\gamma)/\mathrm{d}q=0$. From the first-order condition (3), we have

$$p_1-c-p_1F(q^\gamma)+\int_{\hat{\xi}(q^\gamma)}^{\tilde{\xi}(q^\gamma)}\frac{\partial}{\partial q}(I(q^\gamma,\xi)p_2(I(q^\gamma,\xi),\xi)\mathrm{d}F(\xi)+\int_{\tilde{\xi}(q^\gamma)}^{q^\gamma}p_1\,\mathrm{d}F(\xi)=0,\quad\text{i.e.,}$$

$$p_1F(q^\gamma)-\int_{\hat{\xi}(q^\gamma)}^{\tilde{\xi}(q^\gamma)}\frac{\partial}{\partial q}(I(q^\gamma,\xi)p_2(I(q^\gamma,\xi),\xi)\mathrm{d}F(\xi)-\int_{\tilde{\xi}(q^\gamma)}^{q^\gamma}p_1\,\mathrm{d}F(\xi)=p_1-c.$$

After replacing the terms $\left(p_1F(q^\gamma)-\int_{\hat{\xi}(q^\gamma)}^{\tilde{\xi}(q^\gamma)}\frac{\partial}{\partial q}(I(q^\gamma,\xi)p_2(I(q^\gamma,\xi),\xi)\mathrm{d}F(\xi)-\int_{\tilde{\xi}(q^\gamma)}^{q^\gamma}p_1\,\mathrm{d}F(\xi)\right)$ with p_1-c, we can rewrite the first-order condition (6) as follows:

$$p_1-w-(1-\gamma_w)\left(p_1F(q^\gamma)-\int_{\hat{\xi}(q^\gamma)}^{\tilde{\xi}(q^\gamma)}\frac{\partial}{\partial q}(I(q^\gamma,\xi)p_2(I(q^\gamma,\xi),\xi))\mathrm{d}F(\xi)-\int_{\tilde{\xi}(q^\gamma)}^{q^\gamma}p_1\,\mathrm{d}F(\xi)\right)=p_1-w-(1-\gamma_w)(p_1-c)=0,$$

$$\text{i.e., }\gamma_w=\frac{w-c}{p_1-c}.$$

(ii) After plugging $w=\gamma_w(p_1-c)+c$ (by Eq. (7)) and $q=q^0$ into the retailer's expected profit function (5) and applying some algebraic manipulations, we get $\Pi_r(q^0,w,\gamma_w)=(1-\gamma_w)\Pi(q^0)$ and $\Pi_m(q^0,w,\gamma_w)=\gamma_w\Pi(q^0)$.

(iii) From (7) we see that if $w=c$, then $\gamma_w=0$ and if $w=p_1$, then $\gamma_w=1$. Since from Proposition 3(iv), γ_w is increasing in w, if we increase w from c to p_1, then the manufacturer's percentage of the total supply chain profit γ_w will increase from 0% to 100%, i.e., such PMM contracts can arbitrarily allocate supply chain profit between the retailer and the manufacturer.

(iv) The proof is straightforward so we omit it.

(v) Observe that $M^*/w = \gamma_w(p_1 - p_2)/w = \frac{(w-c)(p_1-p_2)}{(p_1-c)w} < \frac{(w-c)p_1}{(p_1-c)w} = \frac{p_1w-p_1c}{p_1w-wc}$. Since $p_1 > w$, we have $\frac{p_1w-p_1c}{p_1w-wc} < 1$, which implies $M^*/w < 1$, i.e., $M^* < w$. □

Proposition 3(i)–(iii) altogether imply that the PMM contract with (w,γ_w) coordinates the supply chain and, through alternative choices of w (or γ equivalently), can arbitrarily allocate supply chain profit between the manufacturer and retailer. The relationship among the PMM coordinating contract parameters is simple and suitable for interpretations that, while clear in hindsight from the expressions, may not be obvious before-hand. From (7), the set of PMM coordinating parameters can be expressed in terms of either wholesale price or markdown money percentage:

$$w \text{ and } \gamma_w = (w-c)/(p_1-c), \text{ or equivalently, } \gamma \text{ and } w_\gamma = c + \gamma(p_1 - c).$$

Interestingly, the manufacturer's percentage of the supply chain profit is exactly the same as the markdown money percentage. Thus, a higher coordinating markdown money percentage will result in a higher (lower) manufacturer's (retailer's) percentage of the supply chain profit. At the first glance, this result seems counterintuitive since a higher markdown money percentage means a larger markdown money payment from the manufacturer to the retailer. However, from (7), we see that a higher coordinating markdown money percentage also means that the manufacturer charges a higher wholesale price to the retailer, which results in a higher manufacturer profit from the retailer's order to offset the markdown money payment.

Let $\text{Exp}[\Pi]$ and $\text{Var}[\Pi]$ be the mean and variance of the supply chain's profit. (I made this change because Π was used for supply chain profit above.) Then the mean and variance of the manufacturer's profit under the coordinating PMM contract (w,γ_w) can be expressed as $\gamma_w\text{Exp}[\Pi]$ and $\gamma_w^2\text{Var}[\Pi]$, respectively. Similarly, the mean and variance of the retailer's profit under the coordinating PMM contract (w,γ_w) can be expressed as $(1-\gamma_w)\text{Exp}[\Pi]$ and $(1-\gamma_w)^2\text{Var}[\Pi]$, respectively. The expressions highlight a clear risk-return relationship in the PMM contract: as γ_w increases, both the expected profit and the risk from uncertain demand in term of the variance of profit shifts from the retailer to the manufacturer.

The particular profit split and allocation of supply chain risk may depend upon the firms' relative bargaining power. If the manufacturer (retailer) is more powerful and would like to coordinate the supply chain, then one can expect a high (low) markdown money percentage γ_w in the PMM contract. The value of γ_w raises another issue in the PMM contract. If γ_w is very high (e.g., close to 1), then the retailer's expected profit is quite small (e.g., nearly zero). Thus, in absolute terms, a deviation from the supply chain optimal order quantity q^0 imposes little penalty on the retailer but has significant effect on the manufacturer's profit. Under this situation, the coordinating PMM contract may not be enforceable especially if the retailer feels slighted and orders a suboptimal quantity in order to retaliate against the manufacturer. On the other hand, if γ_w is small, then a deviation from the supply chain optimal order quantity q^0 imposes large penalty on the retailer but has less of an effect on the manufacturer's profit. Under this situation, the coordinating PMM contract is more enforceable, i.e., there is no incentive for the retailer to retaliate against the manufacturer by ordering a suboptimal quantity.

Note that the channel coordinating PMM contract parameters are independent of both regular season demand distribution and the clearance revenue function (see (7)). Consequently, a manufacturer can offer a single PMM contract to multiple non-competing retailers with the same regular selling price but different demand distributions and clearance revenue functions. This property is helpful for avoiding antitrust issues that may arise when contract terms vary by customer. For example, Kirkpatrick (2001) reports that independent bookstores have accused Barnes & Noble and Borders of striking preferential deals with publishers that include more generous returns policies.

Proposition 3(iv) is intuitive and says that the higher the wholesale price, the higher the channel coordinating markdown money percentage; the higher the retail price and production cost, the lower the channel coordinating markdown money percentage. We should note that the manufacturer's actual markdown money M^* paid to the retailer for each unit of leftover inventory depends upon the realized demand in the regular season and the retailer's actual clearance price. If realized demand in the regular season is too low so that the leftover inventory is too high, then the retailer has to liquidate overstock at a low clearance price. This will result in higher manufacturer's markdown money. However, as Proposition 3(v) shows, the actual unit markdown money M^* can never be higher than the wholesale price w.

3.3. Decentralized supply chain with a QMM contract

In this section, we investigate the role of the QMM contract on the supply chain coordination. The QMM contract (w,m) specifies that the manufacturer charges the retailer a unit wholesale price w and pays the retailer markdown money $m < w$ for each unsold unit after the regular season but at a lower clearance price (and salvage value) in T_2. As noted above, the QMM contract is essentially a returns policy since it is the retailer whole salvage the unsold products from T_1 at a clearance price in T_2.

Similarly, we can express the retailer's revenue function, $R_2(q,m,\xi)$, given realized demand ξ in T_1, as follows:

$$R_2(q,m,\xi) = \begin{cases} \hat{s}_2(\xi)p_2(\hat{s}_2(\xi),\xi) + mI(q,\xi), & 0 \leqslant \xi \leqslant \hat{\xi}(q), \\ I(q,\xi)[p_2(I(q,\xi),\xi) + m], & \hat{\xi}(q) < \xi \leqslant \tilde{\xi}(q), \\ I(q,\xi)(p_1 + m), & \tilde{\xi}(q) < \xi \leqslant q, \\ 0, & q < \xi. \end{cases}$$

Then, we can express the retailer's expected profit under the QMM contract as follows:

$$\Pi_r(q,w,m) = -wq + R_1(q) + R_2(q,m), \tag{8}$$

where

$$R_2(q,m) = \int_0^{\hat{\xi}(q)} \hat{s}_2(\xi)p_2(\hat{s}_2(\xi),\xi)\mathrm{d}F(\xi) + \int_{\hat{\xi}(q)}^{\tilde{\xi}(q)} I(q,\xi)p_2(I(q,\xi),\xi)\mathrm{d}F(\xi) + \int_{\tilde{\xi}(q)}^{q} p_1 I(q,\xi)\mathrm{d}F(\xi) + \int_0^q mI(q,\xi)\mathrm{d}F(\xi).$$

Proposition 4. *The retailer's profit-maximizing optimal order quantity is determined by some q^m that solves the following first-order condition:*

$$p_1 - w - (p_1 - m)F(q^m) + \int_{\hat{\xi}(q^m)}^{\tilde{\xi}(q^m)} \frac{\partial}{\partial q}(I(q^m,\xi)p_2(I(q^m,\xi),\xi))\mathrm{d}F(\xi) + \int_{\tilde{\xi}(q^m)}^{q^m} p_1\,\mathrm{d}F(\xi) = 0. \tag{9}$$

Proof. After taking the first-derivative of $\Pi_r(q,w,m)$ expressed in (8) with respect to q, we get

$$\mathrm{d}\Pi_r(q,w,m)/\mathrm{d}q = p_1 - w - (p_1 - m)F(q) + \int_{\tilde{\xi}(q)}^{\bar{\xi}(q)} \frac{\partial}{\partial q}(I(q,\xi)p_2(I(q,\xi),\xi))\mathrm{d}F(\xi) + \int_{\bar{\xi}(q)}^{q} p_1\,\mathrm{d}F(\xi).$$

Therefore, the optimal order quantity q^m must satisfy $\mathrm{d}\Pi_r(q,w,m)/\mathrm{d}q = 0$, i.e., Eq. (9). □

We wish to note that compared with the PMM contract which can always coordinate the supply chain, the QMM contract is more restrictive and may not be able to coordinate the supply chain if the solution to the first-order condition (9) is not unique. We next focus only on the coordinating QMM contract when (9) has a unique solution q^m.

Proposition 5. *If q^m is unique, then consider the set of QMM contracts with*

$$m_w = (w - c)/F(q^0). \tag{10}$$

(i) *The retailer orders the integrated supply chain's optimal order quantity q^0, i.e., those contracts can coordinate the supply chain.*
(ii) *The manufacturer's expected profit $\Pi_m(q^0, w, m_w)$ is increasing in w and the retailer's expected profit $\Pi_r(q^0, w, m_w)$ is decreasing in w.*
(iii) *The set of QMM contracts (w, m_w) can arbitrarily allocate integrated supply chain profit between the manufacturer and the retailer.*
(iv) *m_w is increasing in w and decreasing in p_1 but may increase or decrease in c.*
(v) *$m_w < w$.*

Proof. (i) The proof is similar to Proposition 3(i) so we omit it.

(ii) The manufacturer's expected profit under the coordinating QMM contract (w, m_w) is

$$\Pi_m(q^0, w, m_w) = (w - c)q^0 - m_w \int_0^{q^0} I(q^0,\xi)\mathrm{d}F(\xi). \tag{11}$$

After taking the first-derivative of $\Pi_m(q^0,w,m_w)$ with respect to w, from (11) we get

$$\mathrm{d}\Pi_m(q^0,w,m_w)/\mathrm{d}w = q^0 - \frac{1}{F(q^0)}\int_0^{q^0} I(q^0,\xi)\mathrm{d}F(\xi) > 0 \tag{12}$$

and expression (12) also implies $\mathrm{d}\Pi_r(q^0,w,m_w)/\mathrm{dit\ w} < 0$.

(iii) From (10), if $w = c$, then $m_w = 0$ and $\Pi_m(q^0,w,m_w) = 0$; if $w = p_1$, then since $m_w < w = p_1$ by Proposition 5(v), we have $\Pi_r(q^0,w,m_w) < 0$ and $\Pi_m(q^0,w,m_w) > \Pi(q^0)$. Since from Proposition 5(ii), $\mathrm{d}\Pi_m(q^0,w,m_w)/\mathrm{d}w > 0$, there must exist a wholesale price $w_T < p_1$ such that $\Pi_m(q^0,w,m_w) = \Pi(q^0)$. Thus, the set of contracts specified in (10) can arbitrarily allocate supply chain profit between the manufacturer and the retailer.

(iv) It follows from (10) that $\mathrm{d}m_w/\mathrm{d}w = 1/F(q^0) > 0$. Observe from (3) that $\mathrm{d}q^0/\mathrm{d}p_1 > 0$, it follows that $\mathrm{d}m_w/\mathrm{d}p_1 = \frac{-(w-c)f(q^0)}{[F(q^0)]^2}\left(\frac{\mathrm{d}q^0}{\mathrm{d}p_1}\right) < 0$. Similarly, since $\mathrm{d}q^0/\mathrm{d}c < 0$, the sign of $\mathrm{d}m_w/\mathrm{d}c = \frac{-F(q^0)-(w-c)f(q^0)\left(\frac{\mathrm{d}q^0}{\mathrm{d}c}\right)}{[F(q^0)]^2}$ can be positive or negative.

(v) Since the term $\int_{\tilde{\xi}(q^0)}^{\bar{\xi}(q^0)} \frac{\partial}{\partial q}(I(q^0,\xi)p_2(I(q^0,\xi),\xi)\mathrm{d}F(\xi) + \int_{\bar{\xi}(q^0)}^{q^0} p_1\,\mathrm{d}F(\xi)$ in (3) is strictly positive, we have $F(q^0) > (p_1 - c)/p_1 > (w - c)/w$. Therefore, $m_w = (w - c)/F(Q^*) < w$. □

If the retailer's optimal order quantity under the QMM contract is unique, then Proposition 5(i)–(iii) altogether imply that the QMM contract with (w,m_w) coordinates the supply chain and, through alternative choices of w (or m equivalently), can arbitrarily allocate supply chain profit between the manufacturer and retailer. The relationship among the QMM coordinating contract parameters is not as simple as the PMM contract, but can still be expressed in terms of either wholesale price or markdown money:

$$w \text{ and } m_w = (w - c)/F(q^0), \text{ or equivalently}, m \text{ and } w_m = mF(q^0) + c.$$

The effect of increases in the contract parameters w and m_w is similar to the effect of increases in w and γ_w in a PMM contract; the manufacturer's share of total expected supply chain profit is increasing in w and γ_w, and the retailer's share of total expected supply chain profit is decreasing in w and γ_w. Accordingly, we see the same type of risk-return relationship that arises in a QMM contract: as w and m_w increase, both the expected profit and the risk from uncertain demand shifts from the retailer to the manufacturer.

Compared with the PMM contract, we see from (10) that the optimal markdown money in a QMM contract depends upon the regular season demand distribution. For a fixed q^0, a higher probability of stocking out will lead to higher markdown money paid to the retailer. Thus, one disadvantage of the QMM contract is that the manufacturer cannot offer a uniform channel coordinating QMM contract to multiple non-competing retailers.

Proposition 5(iv) and (v) describes relationships among the optimal markdown money and other price/cost parameters, i.e., markdown money is increasing in wholesale price, decreasing in regular retail price, and less than the wholesale price.

4. Supply chain models with a linear clearance demand function

To gain additional insight, we next investigate a supply chain with a linear clearance demand function. We assume clearance demand $D_2(p_2,\xi)$ in T_2 is a combination of a linear demand function $d_2(p_2)$ and a multiplicative shock $x(\xi)$, i.e.,

$$D_2(p_2,\xi) = d_2(p_2)x(\xi), \tag{13}$$

where $d_2(p_2)$ is in the form of

$$d_2(p_2) = a - bp_2, \tag{14}$$

with $a > 0$ and $b \geqslant 0$. The parameter a represents the size of the clearance market and b represents the price sensitivity of clearance demand. The linear demand curve $d_2(p_2)$ is common in the marketing and supply chain literature and has empirical support (e.g., Lilien et al., 1992; Monroe, 1990; Trivedi, 1998; Corbett et al., 2004). Similar to Cachon and Kök (2007), we assume the multiplicative shock function $x(\xi) = \{\mu, \xi\}$, where μ is the mean demand in T_1. The case of $x(\xi) = \mu$ means that demands in the regular season and clearance period are independent and appeal to two distinct market segments, e.g., the firm practices clearance pricing through a separate channel such as an online website (e.g., J.C. Penny and Gap) or a discount specialist (e.g., T.J. Maxx in the apparel industry). The case of $x(\xi) = \xi$ means that regular season and clearance period demands are positively correlated, i.e., higher regular season demand is a good indicator of higher clearance demand.

From (13) and (14), the inverse demand function is

$$p_2(s_2,\xi) = \frac{1}{b}\left(a - \frac{s_2}{x(\xi)}\right), \tag{15}$$

where s_2 is the clearance demand inT_2. In addition, we see that $D_2(p_2,\xi) = 0\ \forall\ p_2 \geqslant p_{\max} = a/b$ and $D_2(p_2,\xi) = ax(\xi)$ if $p_2 = 0$. We assume $p_{\max} \leqslant p_1$, i.e., $a/b \leqslant p_1$, so that $\forall s_2 > 0$, $p_2(s_2, \xi) < p_1$. This assumption essentially captures the characteristic of perishable goods, i.e., the product will lose value in the eyes of the customer after the regular selling season.

It follows from (15) that $\mathrm{d}p_2(s_2,\xi)/\mathrm{d}a > 0$ and $dp_2(s_2,\xi)/\mathrm{d}b < 0$, i.e., the larger the clearance demand pool a and the smaller the price sensitivity b, the higher the clearance price. Let

$$\widehat{R}_2(s_2,\xi) = s_2 p_2(s_2,\xi) = \frac{1}{b}\left(as_2 - \frac{s_2^2}{x(\xi)}\right) \tag{16}$$

be the unconstrained revenue function. Since $\widehat{R}_2(s_2,\xi)$ is quadratic, there exists a unique optimal sales quantity $\hat{s}_2(\xi) = ax(\xi)/2$ in T_2 that maximizes $\widehat{R}_2(s_2,\xi)$, and the corresponding maximal clearance revenue is $\widehat{R}_2(\hat{s}_2,\xi) = a^2x(\xi)/4b$.

Let $\hat{\xi}(q) = \max\{0,\xi_0\}$, where ξ_0 is uniquely determined by solving $q-\xi_0 = \hat{s}_2(\xi_0)$. If realized demand ξ in T_1 satisfies $\xi > \hat{\xi}(q)$, then the leftover over inventory in T_1 must be less than the optimal sales quantity in T_2, i.e., $q-\xi < \hat{s}_2(\xi)$, and the retailer will liquidate all $(q-\xi)$ units of leftover inventory via clearance pricing. If $\xi < \hat{\xi}(q)$, then $q-\xi > \hat{s}_2(\xi)$, and the retailer will only liquidate $\hat{s}_2(\xi)$ units of leftover inventory via clearance pricing. Therefore, we can express the clearance revenue function $R_2(q,\xi)$ in T_2 as follows:

$$R_2(q,\xi) = \begin{cases} \frac{1}{b}\left(a(q-\xi) - \frac{(q-\xi)^2}{x(\xi)}\right) & \text{if } \xi > \hat{\xi}(q), \\ \frac{a^2x(\xi)}{4b} & \text{otherwise.} \end{cases} \tag{17}$$

From (17), we can express expected profit functions of the integrated firm and the independent retailer under the wholesale price-only contract as follows:

$$\begin{aligned}\Pi(q) &= (p_1-c)q - p_1I(q) + \int_0^{\hat{\xi}(q)} \frac{a^2x(\xi)}{4b}\,\mathrm{d}F(\xi) \\ &\quad + \frac{1}{b}\int_{\hat{\xi}(q)}^{q}\left(a(q-\xi) - \frac{(q-\xi)^2}{x(\xi)}\right)\mathrm{d}F(\xi)\mathrm{d}x, \end{aligned} \tag{18}$$

$$\begin{aligned}\Pi_r(q) &= (p_1-w)q - p_1I(q) + \int_0^{\hat{\xi}(q)} \frac{a^2x(\xi)}{4b}\,\mathrm{d}F(\xi) \\ &\quad + \frac{1}{b}\int_{\hat{\xi}(q)}^{q}\left(a(q-\xi) - \frac{(q-\xi)^2}{x(\xi)}\right)\mathrm{d}F(\xi)\mathrm{d}x. \end{aligned} \tag{19}$$

Proposition 6. (i) *With the linear clearance demand function, the integrated firm's optimal stocking level q^0 and the independent retailer's optimal order quantity q^r are unique and satisfy the following first-order conditions:*

$$p_1 - c - p_1F(q^0) + \frac{1}{b}\int_{\hat{\xi}(q^0)}^{q^0}\left(a - \frac{2(q^0-\xi)}{x(\xi)}\right)\mathrm{d}F(\xi) = 0, \tag{20}$$

$$p_1 - w - p_1F(q^r) + \frac{1}{b}\int_{\hat{\xi}(q^r)}^{q^r}\left(a - \frac{2(q^r-\xi)}{x(\xi)}\right)\mathrm{d}F(\xi) = 0. \tag{21}$$

(ii) *Both q^0 and q^r are increasing in a and decreasing in b.*

Proof.

(i) After taking the first derivative of (18) with respect to q, we get:

$$\begin{aligned}\frac{\mathrm{d}\Pi(q)}{\mathrm{d}q} &= p_1 - c - p_1F(q) \\ &\quad + \frac{1}{b}\left(\frac{a^2x(\hat{\xi}(q))}{4} - a(q-\hat{\xi}(q)) + \frac{(q-\hat{\xi}(q))^2}{x(\hat{\xi}(q))}\right)f(\hat{\xi}(q))\hat{\xi}'(q) \\ &\quad + \frac{1}{b}\int_{\hat{\xi}(q)}^{q}\left(a - \frac{2(q-\xi)}{x(\xi)}\right)\mathrm{d}F(\xi). \end{aligned} \tag{22}$$

Recall that $\hat{\xi}(q) = \max\{0,\xi_0\}$. If $\hat{\xi}(q)=0$, then $\hat{\xi}'(q)=0$. If $\hat{\xi}(q)=\xi_0$, then since $q-\xi_0 = \hat{s}_2(\xi_0) = ax(\xi_0)/2$, we have

$$\frac{a^2x(\hat{\xi}(q))}{4} - a(q-\hat{\xi}(q)) + \frac{(q-\hat{\xi}(q))^2}{x(\hat{\xi}(q))} = 0.$$

Therefore, for any $\hat{\xi}(q) = \max\{0,\xi_0\}$, we can rewrite (21) as follows:

$$\frac{\mathrm{d}\Pi(q)}{\mathrm{d}q} = (p_1-c) - p_1F(q) + \frac{1}{b}\int_{\hat{\xi}(q)}^{q}\left(a - \frac{2(q-\xi)}{x(\xi)}\right)\mathrm{d}F(\xi). \tag{23}$$

After taking the second derivative of (23) with respect to q, we get:

$$\begin{aligned}\frac{\mathrm{d}^2\Pi(q)}{\mathrm{d}q^2} &= -\left(p_1 - \frac{a}{b}\right)f(q) - \frac{2}{b}\int_{\hat{\xi}(q)}^{q}\frac{1}{x(\xi)}\mathrm{d}F(\xi) \\ &\quad - \frac{1}{b}\left(a - \frac{2(q-\hat{\xi}(q))}{x(\hat{\xi}(q))}\right)f(\hat{\xi}(q))\hat{\xi}'(q). \end{aligned} \tag{24}$$

Similarly, If $\hat{\xi}(q)=0$, then $\hat{\xi}'(q)=0$. If $\hat{\xi}(q)=\xi_0$, then $a - \frac{2(q-\hat{\xi}(q))}{x(\hat{\xi}(q))} = 0$. By the assumption of $a \leqslant bp_1$, we must have

$$\frac{\mathrm{d}^2\Pi(q)}{\mathrm{d}q^2} = -\left(p_1 - \frac{a}{b}\right)f(q) - \frac{2}{b}\int_{\hat{\xi}(q)}^{q}\frac{1}{x(\xi)}\,\mathrm{d}F(\xi) < 0,$$

which implies $\Pi(q)$ is concave. Therefore, the optimal order quantity q^0 must be uniquely determined by the first-order condition (20). Similarly, the independent retailer's optimal order quantity q^r must be uniquely determined by the first-order condition (21).

(ii) By the Implicit Function Theorem, from (20), we have

$$\mathrm{d}q^0/\mathrm{d}a = \frac{\mathrm{d}^2\Pi(q^0)/\mathrm{d}q\,\mathrm{d}a}{-\mathrm{d}^2\Pi(q^0)/\mathrm{d}q^2} = \frac{\frac{1}{b}\int_{\hat{\xi}(q^0)}^{q^0}\mathrm{d}F(\xi)}{-\mathrm{d}^2\Pi(q^0)/\mathrm{d}q^2} > 0,$$

$$\mathrm{d}q^0/\mathrm{d}b = \frac{\mathrm{d}^2\Pi(q^0)/\mathrm{d}q\,\mathrm{d}b}{-\mathrm{d}^2\Pi(q^0)/\mathrm{d}q^2} = \frac{-\frac{1}{b^2}\int_{\hat{\xi}(q^0)}^{q^0}\left(a - \frac{2(q^0-\xi)}{x(\xi)}\right)\mathrm{d}F(\xi)}{-\mathrm{d}^2\Pi(q^0)/\mathrm{d}q^2} < 0.$$

Similarly, we can prove $\mathrm{d}q^r/\mathrm{d}a > 0$ and $\mathrm{d}q^r/\mathrm{d}b < 0$.

Proposition 6(i) characterizes the integrated supply chain's optimal stocking level q^0 and the independent retailer's optimal order quantity q^r under the linear clearance demand function and Proposition 6(ii) says that the larger the size of the clearance market, the higher the integrated firm's optimal inventory stocking level and the independent retailer's optimal order quantity; the more price-sensitive the clearance demand is, the lower the integrated firm's optimal inventory stocking level and the independent retailer's optimal order quantity.

We next investigate how the linear demand parameters (a,b) interact with the channel coordinating PMM and QMM contracts. From Proposition 3, we know that the channel coordinating PMM contract parameter is independent of both a and b in the linear clearance demand function. However, from Proposition 5, we know that the channel coordinating QMM contract depends upon the clearance demand parameters. The nature of this dependency is shown in the following proposition. □

Proposition 7. *The coordinating markdown money m_w in the QMM contract is decreasing in a and increasing in b.*

Proof. Recall from (10) that $m_w = (w-c)/F(q^0)$. From Proposition 6(ii), we know $\mathrm{d}q^0/\mathrm{d}a > 0$ and $\mathrm{d}q^0/\mathrm{d}b < 0$. Therefore, we get: $\frac{\mathrm{d}m_w}{\mathrm{d}a} = -\frac{(w-c)f(q^0)}{[F(q^0)]^2}\left(\frac{\mathrm{d}q^0}{\mathrm{d}a}\right) < 0$ and $\frac{\mathrm{d}m_w}{\mathrm{d}b} = -\frac{(w-c)f(q^0)}{[F(q^0)]^2}\left(\frac{\mathrm{d}q^0}{\mathrm{d}b}\right) > 0$. □

Proposition 7 says that if the QMM contract can coordinate the supply chain, then the larger the clearance market, the less the manufacturer's markdown money to coordinate the supply chain. In addition, the more price-sensitive the clearance demand is, the larger the manufacturer's markdown money. We next express the manufacturer's expected profit functions under the coordinating PMM and QMM contract as follows:

$$\Pi_m(q^0, w, \gamma_w) = (w-c)q^0 - \gamma_w p_1 I(q^0) + \frac{a^2\gamma_w}{4b}\int_0^{\hat{\xi}(q^0)} x(\xi)\mathrm{d}F(\xi) + \frac{\gamma_w}{b}\int_{\hat{\xi}(q^0)}^{q^0}\left(a(q^0-\xi) - \frac{(q^0-\xi)^2}{x(\xi)}\right)\mathrm{d}F(\xi), \quad (25)$$

$$\Pi_m(q^0, w, m_w) = (w-c)q^0 - m_w I(q^0). \quad (26)$$

In practice, it can sometimes be difficult for the manufacturer to change the wholesale price for the following reasons: (1) if the market is highly competitive, then the manufacturer does not have much control of the market price and has to act as a price-taker; (2) the Robinson–Patman Act in the US restricts the manufacturer's ability to sell the same product to different retailers at different wholesale prices, especially when the production cost is the same; and (3) changing the wholesale price can be costly, and there is empirical evidence that manufacturers are reluctant to change wholesale prices (e.g., Iyer and Bergen, 1997 and Cachon, 2003). Bosh and Anand (2007) also provide additional support for exogenous wholesale price in practice. In view of this, our next proposition compares the relative performance of the coordinating PMM and QMM contracts, when demands in T_1 and T_2 are independent, i.e., $x(\xi) = \mu$.

Proposition 8. *If $x(\xi) = \mu$ and $\hat{\xi}(q^0) = 0$, then for a fixed w, the manufacturer (retailer)'s expected profit under the coordinating PMM contract is higher (lower) than that under the coordinating QMM contract if and only if $\mathrm{CV}_{I(q^0)} < \sqrt{\frac{2-F(q^0)}{F(q^0)}}$, where $\mathrm{CV}_{I(q^0)}$ is the coefficient of variation of $I(q^0)$.*

Proof. Let $\Delta = \Pi_m(q^0,w,\gamma_w) - \Pi_m(q^0,w,m_w)$ be the difference between the manufacturer's expected profits under the coordinating PMM and QMM contract, and let $\mu_{I(q^0)}$ and $\sigma_{I(q^0)}$ be the mean and standard deviation of the random variable $I(q^0)$, respectively. For a fixed w, if $x(\xi) = \mu$ and $q^0 \leqslant a\mu/2$, then $\hat{\xi}(q) = 0$. From (25) and (26), we have

$$\Delta = \left(m_w - \gamma_w p_1 + \frac{a\gamma_w}{b}\right)\mu_{I(q^0)} - \frac{\gamma_w}{b\mu}\int_0^{q^0} I(q^0,\xi)^2\,\mathrm{d}F(\xi), \quad (27)$$

Since $\sigma^2_{I(q^0)} = \int_0^{q^0} I(q^0,\xi)^2\,\mathrm{d}F(\xi) - \mu^2_{I(q^0)}$, we can rewrite (27) as follows:

$$\Delta = \left(m_w - \gamma_w p_1 + \frac{a\gamma_w}{b}\right)\mu_{I(q^0)} - \frac{\gamma_w}{b\mu}\left(\sigma^2_{I(q^0)} + \mu^2_{I(q^0)}\right) = \mu^2_{I(q^0)}\left(\frac{w-c}{p_1-c}\right)\left(\frac{p_1 - c - (p_1 - \frac{a}{b})F(q^0)}{\mu_{I(q^0)}F(q^0)} - \frac{\mathrm{CV}^2_{I(q^0)}+1}{b\mu}\right). \quad (28)$$

From (20) we see that

$$p_1 - c - \left(p_1 - \frac{a}{b}\right)F(q^0) = \frac{2}{b\mu}\mu_{I(q^0)}.$$

Therefore, we can rewrite (28) as follows:

$$\Delta = \left(\frac{(w-c)\mu^2_{I(q^0)}}{b\mu(p_1-c)}\right)\left(\frac{2}{F(q^0)} - \mathrm{CV}^2_{I(q^0)} - 1\right). \quad (29)$$

It follows from (29) that if $\mathrm{CV}_{I(q^0)} < \sqrt{\frac{2-F(q^0)}{F(q^0)}}$, then $\Delta > 0$, otherwise, $\Delta \leqslant 0$. □

Recall that the condition $x(\xi) = \mu$ means that demands in T_1 and T_2 are independent and the condition $\hat{\xi}(q^0) = 0$ means that the retailer's leftover inventory from T_1 is small enough, i.e., at least less than the unconstrained optimal clearance sales quantity $\hat{s}_2(\xi)$, so that inventory hold backs will not occur. If both conditions hold, then Proposition 8 identifies a necessary and sufficient condition under which the coordinating PMM contract will result in a higher (lower) manufacturer (retailer) profit than the coordinating QMM contract when the wholesale price is fixed.

Since the analytical result on the relative profits associated with PMM and QMM contracts in Proposition 8 applies when the regular season and clearance period demands are independent (i.e., $x(\xi) = \mu$) it would be interesting to compare and contrast results across environments with independent and correlated demands. Accordingly, we conduct a numerical study based upon the following combinations of parameters with a total of 168 scenarios:

- Product profit margin: $m_I = (p_1 - c)/p_1 = \{0.25, 0.50\}$
- Size of the clearance market: $a = \{5, 50\}$
- Maximal clearance price: $p_{\max} = a/b = \{0.2p_1, 0.6p_1, p_1\}$
- Random shock function: $x(\xi) = \{\mu, \xi\}$
- Regular season demand distribution: $D_1 \sim$ [Uniform, Normal, Gamma]

In each scenario, we fix the production cost $c = \$10$ and the mean regular season demand $\mu = 50$.

For the normal distribution, we select a coefficient of variation $\mathrm{CV} = \sigma/\mu = \{0.1, 0.2, 0.3\}$. For the uniform distribution, we choose $D_1 \sim$ Uniform [0,100] with $\mathrm{CV} \approx 0.577$. For the gamma distribution, we select a $\mathrm{CV} = \{0.25, 0.71, 1\}$. For the case of $\mathrm{CV} = 1$, we set the α parameter of the gamma distribution at $\alpha = 1$, which reduces to the exponential distribution. From Proposition 3 we know the PMM contract can always coordinate the supply chain. Our numerical results also show that the retailer's expected profit function under the channel coordinating QMM contract (w, m_w) is unimodal and the retailer's optimal order quantity q^m satisfies $\mathrm{d}\Pi_r(q^m, w, m_w)/\mathrm{d}q = \mathrm{d}\Pi(q^m)/\mathrm{d}q = 0$, i.e., $q^m = q^0$. Therefore, the QMM contract can also coordinate the supply chain in our numerical study. We compute the percentage change in the manufacturer's expected profit under the coordinating PMM contract relative to that under the coordinating QMM contract, i.e.

$$\theta = \frac{\Pi_m(q^0, w, \gamma_w) - \Pi_m(q^0, w, m_w)}{\Pi_m(q^0, w, m_w)} \times 100\%. \quad (30)$$

After plugging (25) and (26) into (30), and noting that the coordinating PMM and QMM contracts parameters satisfy $m_w = (w-c)/F(q^0)$ and $\gamma_w = (w-c)/(p_1-c)$, we rewrite (30) as follows:

$$\theta = \frac{\frac{I(q^0)}{F(q^0)} - \frac{p_1 I(q^0)}{p_1 - c} + \frac{a^2\int_0^{\hat{\xi}(q^0)} x(\xi)\mathrm{d}F(\xi)}{4b(p_1-c)} + \frac{\int_{\hat{\xi}(q^0)}^{q^0}\left(a(q^0-\xi) - \frac{(q^0-\xi)^2}{x(\xi)}\right)\mathrm{d}F(\xi)}{b(p_1-c)}}{q^0 - \frac{I(q^0)}{F(q^0)}} \times 100\%. \quad (31)$$

Interestingly, from (31) we see that θ is independent of the wholesale price w. Our numerical results are reported in Tables 1–3 and rounded up to two decimals.

From Tables 1–3, we find that the coordinating PMM contract results in higher manufacturer expected profits than the coordinating QMM contract for all combinations of the parameter values. This result suggests that for some commonly used distributions such as uniform, normal, gamma, and exponential, the manufacturer will prefer a coordinating PMM contract to a coordinating QMM contract when demands in two periods are independent and correlated.

Our numerical results in Tables 1–3 further show that the manufacturer's expected profit is significantly higher under the PMM contract when (1) the clearance market size a is relatively large and the price sensitivity of the clearance demand b is relatively small, or equivalently, when the maximal clearance price $p_{\max} = a/b$ is relatively large, and (2) the product profit margin m_I is relatively high. For example, when $a = 50$, $p_{\max} = p_1$, and $m_I = 0.50$, the manufacturer's expected profits under the coordinating PMM contract are about 200–600% higher than that under the coordinating QMM contract, for all combinations of other parameter values. However, the differences in the manufacturer's

Table 1
Percentage change in manufacturer expected profit (θ) under the coordinating PMM contract relative to under the coordinating QMM contract when D_1 is uniformly distributed over [0,100] (CV = 0.577)

m_I	a	b	θ (%)	
			$x(\xi) = \mu$	$x(\xi) = \xi$
0.25	5	1.875	0.98	0.00%
		0.625	7.50	0.00
		0.375	33.33	0.00
	50	18.75	0.10	0.00
		6.25	1.16	0.00
		3.75	309.83	95.93
0.50	5	1.875	1.86	0.00
		0.625	11.11	0.00
		0.375	49.17	0.00
	50	18.75	0.21	0.00
		6.25	40.87	5.61
		3.75	623.67	346.59

Table 2
Percentage change in manufacturer expected profit (θ) under the coordinating PMM contract relative to under the coordinating QMM contract when D_1 is normally distributed

m_I	CV	a	b	θ (%)	
				$x(\xi) = \mu$	$x(\xi) = \xi$
0.25	0.1	5	1.875	0.01	0.01
			0.625	0.11	0.11
			0.375	30.45	29.09
		50	18.75	0.00	0.00
			6.25	0.01	0.01
			3.75	312.42	308.21
	0.2	5	1.875	0.04	0.02
			0.625	0.43	0.33
			0.375	28.13	22.19
		50	18.75	0.00	0.00
			6.25	0.06	0.05
			3.75	312.18	293.96
	0.3	5	1.875	0.12	0.00
			0.625	1.02	0.04
			0.375	25.90	12.92
		50	18.75	0.63	0.00
			6.25	1.61	0.02
			3.75	312.93	263.19
0.50	0.1	5	1.875	0.02	0.02
			0.625	3.93	3.68
			0.375	62.10	60.43
		50	18.75	0.00	0.00
			6.25	41.64	41.01
			3.75	624.96	617.55
	0.2	5	1.875	0.09	0.08
			0.625	3.87	3.08
			0.375	60.90	53.92
		50	18.75	0.01	0.01
			6.25	41.57	38.82
			3.75	624.84	593.23
	0.3	5	1.875	0.22	0.09
			0.625	4.83	2.91
			0.375	58.91	43.77
		50	18.75	0.27	0.01
			6.25	41.81	34.16
			3.75	624.92	550.49

Table 3
Percentage change in manufacturer expected profit (θ) under the coordinating PMM contract relative to under the coordinating QMM contract when D_1 follows the gamma distribution

m_I	CV	a	b	θ (%)	
				$x(\xi) = \mu$	$x(\xi) = \xi$
0.25	0.25	5	1.875	0.05	0.05
			0.625	0.66	0.64
			0.375	27.68	20.52
		50	18.75	0.01	0.01
			6.25	0.09	0.09
			3.75	311.99	286.21
	0.71	5	1.875	0.44	0.34
			0.625	4.74	2.07
			0.375	41.50	8.96
		50	18.75	0.05	0.08
			6.25	0.82	1.15
			3.75	308.53	126.75
	1	5	1.875	0.76	0.29
			0.625	8.91	1.49
			0.375	62.47	4.72
		50	18.75	0.08	0.16
			6.25	1.60	1.86
			3.75	306.61	48.35
0.50	0.25	5	1.875	0.00	0.15
			0.625	4.86	3.99
			0.375	60.01	50.20
		50	18.75	0.02	0.02
			6.25	41.52	37.66
			3.75	624.75	579.62
	0.71	5	1.875	1.27	0.86
			0.625	14.96	5.64
			0.375	64.92	21.68
		50	18.75	0.14	0.25
			6.25	40.69	16.11
			3.75	623.05	347.93
	1	5	1.875	2.51	0.84
			0.625	24.86	4.41
			0.375	84.33	13.54
		50	18.75	0.30	0.54
			6.25	42.82	11.59
			3.75	621.05	199.05

expected profits under the coordinating PMM and QMM contracts are insignificant when (1) the clearance market size a is relatively small and the price sensitivity of the clearance demand b is relatively large, or equivalently, when the maximal clearance price $p_{\max}$ is relatively small, and (2) the product profit margin m_I is relatively low,. For example, when $a = 5$, $p_{\max} = 0.2p_1$, and $m_I = 0.25$, the percentage increases in the manufacturer's expected profits under the coordinating PMM contract relative to that under the coordinating QMM contract are all less than 1% for all combinations of other parameter values.

Such results can be explained by reexamining the manufacturer's expected profit functions under the coordinating PMM and QMM contracts in (25) and (26). Although the manufacturer makes the same profit, i.e., $(w-c)q^0$, in the regular season under both coordinating contracts, the manufacturer's profits under the two contracts become different after the regular season. More specifically, under the PMM contract, from (25) we see that the manufacturer not only pays the retailer some money $\gamma_w p_1$ for each unsold unit in the regular season, but also shares γ_w percent of the retailer's clearance revenue, $R_2(q^0) = \frac{a^2}{4b}\int_0^{\hat{\xi}(q^0)} x(\xi)\mathrm{d}F(\xi) + \frac{1}{b}\int_{\hat{\xi}(q^0)}^{q^0}\left(a(q^0-\xi) - \frac{(q^0-\xi)^2}{x(\xi)}\right)\mathrm{d}F(\xi)$. However, under the QMM contract, from (26) we see that the manufacturer only pays the retailer some markdown money m_w for each unsold unit in the regular season, but does not share any of the retailer's clearance revenue. In other words, the PMM contract allows the manufacturer to continue to make some money in the clearance market whereas the QMM contract does not. When the maximal clearance price is relatively low (e.g., $p_{\max} = 0.2p_1$), the difference in the manufacturer expected profits under two contracts is small since the clearance market is less profitable. However, when the maximal clearance price is relatively high, e.g., $p_{\max} = p_1$, the difference in the manufacturer expected profits under two contracts is large since the clearance market is more profitable. Similarly, as the product profit margin m_I becomes higher, the optimal order quantity q^0 also becomes larger. This means that there will be more expected unsold inventory

$I(q^0)$ for clearance sales, which in turn results in a larger clearance revenue and a larger difference between PMM and QMM contract manufacturer profits.

5. Conclusion

The volatile market of perishable products is featured by uncertain demand and a short selling season. It is common in practice that retailers liquidate excess inventory via clearance pricing. In this paper we investigate two forms of markdown money contract schemes, i.e., PMM and QMM, for supply chain coordination. We find that the PMM contract can always coordinate the supply chain and arbitrarily divide the supply chain profit between the manufacturer and the retailer, but the QMM contract may not be able to coordinate the supply chain.

Our results provide some managerial implications on the strengths and limitations of the two forms of markdown money contract schemes for coordinating a supply chain with clearance pricing. First, the QMM contract is more restrictive than the PMM due to the fact that (1) if the retailer's optimal order quantity is not unique, then the QMM contract fails to coordinate the supply chain, and (2) even if a QMM contract can coordinate the supply chain, its parameters depend upon the demand distribution. As pointed out in Cachon (2003), a manufacturer normally sells her product not just to one, but to several retailers and is legally obligated to offer the same contractual terms to their retailers. Hence, it is desirable for the manufacturer to offer a uniform contract to all of her retailers, especially if they only differ in demand. Compared to the QMM contract, the PMM contract can always coordinate the supply chain and its parameters are independent of the demand distribution. Therefore, the manufacturer can offer a uniform PMM contract to multiple retailers.

Second, a PMM coordinating contract has the appealing feature of transparency in allocation of expected supply chain profit. A PMM contract is comprised of two parameters—the wholesale price (w) and the percentage of the price markdown (γ) paid by the manufacturer to the retailer—and the value of γ is the manufacturer's share of expected supply chain profit (and $1-\gamma$ is the retailer's share of expected supply chain profit). In practice, retailers and manufacturers usually have different bargaining powers when they negotiate contracts. For example, the retailers in the fashion industry, e.g., May, Federated, Kohl's, Saks, and J.C. Penney, usually have more bargaining power than the manufacturer, and naturally they would like to have a larger share of the total supply chain profit by selecting a $\gamma < 50\%$ (and $1-\gamma > 50\%$) in the coordinating PMM contract. Furthermore, as related to the point above, supply chain profit allocation under a coordinating PMM contract is independent of the demand distribution, which is not the case under a QMM coordinating contract. Profit allocation transparency and insensitivity to the demand distribution can be useful during contract presentation and negotiation.

Third, in practice, it can sometimes be difficult for the manufacturer to change the wholesale price. Our numerical results based upon a linear clearance demand function suggest that the coordinating PMM contract will generally result in a higher manufacturer's expected profit than the coordinating QMM contract. Therefore, a manufacturer who is inflexible in changing wholesale price should use a PMM contract to coordinate the supply chain and improve her expected profit instead of a QMM contract, especially when the clearance market is highly profitable and the product profit margin is high.

Future research should consider a supply chain comprised of a manufacturer selling to multiple retailers competing in both regular season and clearance period. The relative strengths and limitations of the three forms of supply chain contracts could be tested empirically by experiments or by surveys of managers so that more managerial insights can be obtained into which contract form is preferred to others and why.

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References

Bosh, I., Anand, P., 2007. On returns policies with exogenous price. European Journal of Operational Research 178, 782–788.
Cachon, G.P., 2003. Supply chain coordination with contracts. In: de Kok, A.G., Graves, S.C. (Eds.), Handbook in Operations Research and Management Science, Volume on Supply Chain Management: Design, Coordination and Operation. North Holland, Amsterdam, pp. 229–339.
Cachon, G.P., Kök, A.G., 2007. Implementation of the newsvendor model with clearance pricing: How to (and how not to) estimate a salvage value. Manufacturing and Service Operations Management 9, 276–290.
Corbett, C.J., Zhou, D., Tang, C.S., 2004. Designing supply contracts: Contract type and information asymmetry. Management Science 50, 550–559.
Donohue, K., 2000. Efficient supply contracts for fashion goods with forecast updating and two production modes. Management Science 46, 1397–1411.
Emmons, H., Gilbert, S.M., 1998. Note. The role of returns policies in pricing and inventory decisions for catalogue goods. Management Science 44, 276–283.
Forest, S.A., Zellner, W., Palmer, A.T. 2003. Giving away the store. Business Week, June 2.
Iyer, A., Bergen, M., 1997. Quick response in manufacturer–retailer channels. Management Science 43, 559–570.
Kandel, E., 1996. The right to return. Journal of Law and Economics 39, 329–356.
Khouja, M., 1999. The single-period (news-vendor) problem: Literature review and suggestions for future research. Omega 27, 537–553.
Kirkpatrick, D.D. 2001. Smaller bookstores end court struggle against two chains. New York Times, April 21, A1.
Kratz, E.F., 2005. Marked down. Fortune 152 (4), 103–107.
Krishnan, H., Kapuscinski, R., Butz, D.A., 2004. Coordinating contracts for decentralized supply chains with retailer promotional effort. Management Science 50, 48–63.
Lariviere, M.A., 1999. Supply chain contracting and coordination with stochastic demand. In: Tayur, S., Ganeshan, R., Magazine, M. (Eds.), Quantitative Models for Supply Chain Management. Kluwer Academic Publishers, Boston, MA, pp. 233–268.
Lee, H.L., Nahmias, S., 1993. Single-product, single-location models. In: Graves, S.C., Kan, A.H.G.R., Zipkin, P.H. (Eds.), Handbook in Operations Research and Management Science, Volume on Logistics of Production and Inventory. North Holland, Amsterdam, pp. 3–58.
Lee, H.L., Padmanabhan, V., Taylor, T.A., Whang, S., 2000. Price protection in the personal computer industry. Management Science 46, 467–482.
Lilien, G.L., Kotler, P., Moorthy, K.S., 1992. Marketing Models. Prentice Hall, Englewood Cliffs, NJ.
Monroe, K.B., 1990. Pricing: Making Profitable Decisions, second ed. McGraw Hill, New York, NY.
Padmanabhan, V., Png, I.P.L., 1995. Returns policies: Make money by making good. Sloan Management Review, 65–72. Fall.
Pasternack, B.A., 1985. Optimal pricing and return policies for perishable commodities. Marketing Science 4, 166–176.
Petruzzi, N.C., Dada, M., 1999. Pricing and the newsvendor problem: A review with extensions. Operations Research 47, 183–194.
Porteus, E.L., 1990. Stochastic inventory model. In: Graves, S.C., Kan, A.H.G.R., Zipkin, P.H. (Eds.), Handbook in Operations Research and Management Science, Volume on Stochastic Models. North Holland, Amsterdam, pp. 605–652.
Rozhon, T. 2005. First the markdown, then the showdown. New York Times, February 25.
Spengler, J.J., 1950. Vertical integration and antitrust policy. Journal of Political Economy 58, 347–352.
Su, X., Zhang, F. 2005. Strategic customer behavior, commitment, and supply chain performance. Working paper, UC Berkeley and UC Irvine.
Taylor, T.A., 2002. Supply chain coordination under channel rebates with sales effort effects. Management Science 48, 992–1007.
Trivedi, M., 1998. Distribution channels: An extension of exclusive retailership. Management Science 44, 896–909.
Tsay, A.A., 2001. Managing retail channel overstock: Markdown money and return policies. Journal of Retailing 77, 457–492.
Wang, C.X., Webster, S., 2007. Channel coordination for a supply chain with a risk-neutral manufacturer and a loss-averse retailer. Decision Sciences 38, 361–389.
Webster, S., Weng, Z.K., 2000. A risk-free perishable item returns policy. Manufacturing and Service Operations Management 2, 100–106.