University at Buffalo
The State University of New York

Department of Mathematics

Office of Undergraduate Studies
233 Mathematics Building
E-mail: math-undergrad@buffalo.edu
Ph: (716) 645-8785

SAMPLE SYLLABUS
This document is published as an indication of what is typically taught in this course. Instructors have the responsibility of deciding on topics to be omitted, additional topics to be included and the emphasis, ordering and pacing of presentation.

Course Number: **MTH 311**

Course Title: **Introduction to Higher Mathematics**

Credit Hours: **4.0**

Textbook: ***Outline & Problem Sets,*** **notes developed at the UB Math Department.**

Supplementary Textbook: **G. Chartrand, A. Polimeni, P. Zhang, *Mathematical Proofs: A Transition to Advanced Mathematics,* 3rd edition.**

Prerequisites: MTH 241

Notes: This is a core course in almost all math major concentrations, and usually the first course in higher mathematics that math majors take. Its primary purpose is to teach students how to read, understand, and write rigorous mathematical proofs. It also introduces basic notions in logic and set theory.

This schedule is written for 13 weeks of instruction. In a typical semester there are 14 teaching weeks, thus some flexibility is built in.

Week	Topics
1	Introduction to logic.
2	Axioms for the integers. Divisibility.
3	Axioms for the real numbers.
4	Rational and irrational numbers. **Midterm Exam I**
5	Induction.
6	Sets.
7	Functions. Inverse functions.
8	Cardinality of sets.
9	Countability. **Midterm Exam II**
10	Algebraic and transcendental numbers.
11	Infinite sequences and limits.
12	Least upper bound axiom. Monotone sequence property.
13	Series. **Midterm Exam III**

Student Learning Outcomes for
MTH 311 Introduction to Higher Mathematics

Assessment measures: weekly homework assignments, 3 midterm exams, final exam.

At the end of this course a student will be able to:	Assessment
– recognize statements, determine their truth value, and combine them – recognize implications and operate on them to obtain converse and contrapositive – use universal and existential quantifiers – negate statements and quantifiers – determine when open sentences hold	HW #1 Midterm Exam I Final Exam
– interpret the integer axioms – define well-ordering for integers – prove the additive, multiplicative, and order properties of integers from the axioms – extend well-ordering to bounded sets of integers – write correct direct and indirect proofs	HW #2 Midterm Exam I Final Exam
– analyze the definitions of divisibility and greatest common divisor for integers – extend the properties of even and odd integers to the analogous properties relative to 3 and 10 – write correct proofs using well-ordering	HW #2 Midterm Exam I Final Exam
– contrast the real axioms with the integer axioms – prove the division properties for real numbers – utilize the Archimedean Principle and show equivalence of the Principle to other forms – prove existence of the greatest integer and its properties	HW #3 Midterm Exam I Final Exam
– distinguish rational from irrational numbers – establish properties of sums and products of rational/irrational numbers – recognize why a decimal expansion is repeating if and only if it represents a rational number – show that the square root of 2 is irrational and generalize to other roots and other integers – describe density of a subset of the reals and prove that the rationals and the irrationals are dense – demonstrate that others sets of reals are dense/non-dense	HW #4 Midterm Exam I Final Exam
– analyze the principle of induction in both correct and incorrect usage – use the principle of induction in proofs, including of inequalities and of the Binomial Theorem – analyze the principle of strong induction and use it in proofs – demonstrate equivalence of induction and strong induction	HW #5 Midterm Exam II Final Exam
– recognize and utilize the fundamentals of sets (inclusion, equality, union, intersection, complement, power set) – prove the basic properties of sets, including de Morgan's laws – recognize the formal definition of ordered pair – analyze the Cartesian product and its properties	HW #6 Midterm Exam II Final Exam
– recognize and utilize the definition of a function (as a subset of the Cartesian product), of composition, of image and inverse image – find all functions from a finite set to a finite set – show how to glue functions together to obtain new functions – prove relations between image/inverse image of union and intersections of sets	HW #7 Midterm Exam II Final Exam
– recognize and utilize the definition of one to one function, onto function, restriction of a function to a subset – characterize the identity function – demonstrate that a function has an inverse if and only if it is one to one and onto – find the inverse of a composition of functions – show that if a function satisfies the composition properties of the inverse function then it is the inverse function – demonstrate the relation between one to one functions and inverse image of the image. – prove that gluing together one to one/onto functions on disjoint sets results in one to one/onto functions	HW #7 Midterm Exam II Final Exam

– recognize and utilize the definition of one to one correspondence and equivalence of sets – construct equivalences for various finite sets, intervals, unions and Cartesian products – show that equivalence is reflexive, symmetric, and transitive – distinguish finite from infinite sets – show that subsets and unions of finite sets are finite – prove that the natural numbers are infinite – classify the number of elements in a set	HW #8 Midterm Exam II Final Exam
– differentiate between countable and uncountable sets – show that the set of rational numbers is countable and the set of irrationals is uncountable – prove that subsets, countable unions, and finite Cartesian products of countable sets are countable – show that a set is countable if and only if it can be injected into the natural numbers – show that a set and its power set are never equivalent	HW #9 Midterm Exam II Final Exam
– differentiate between algebraic and transcendental numbers – analyze the steps in showing that the set of algebraic numbers is countable – show that the root of an integer or the sum of two square roots of integers is algebraic	HW #10 Midterm Exam III Final Exam
– verify the properties of absolute value, including the triangle inequality – describe the definition of a sequence – employ the epsilon definition of limit – distinguish between convergent and divergent sequences – prove the limit theorems for sequences and verify that the limit of a sequence of positive terms is non-negative.	HW #11 Midterm Exam III Final Exam
– describe upper/lower bounds and least upper/greatest lower bounds – describe and employ the Least Upper Bound Axiom – prove the Archimedean Principle and the existence of n-th roots – show that the LUB Axiom implies the Greatest Lower Bound Axiom – recognize sup and inf of sets – utilize the LUB/GLB axioms to show that if each element of one non-empty set is less than or equal to each element of another non-empty set then the sup of the first set is less than or equal to the inf of the second	HW #12 Midterm Exam III Final Exam
– show that a convergent sequence is bounded – recognize a monotone sequence and prove the Monotone Sequence Property – *optional: recognize the lim sup and lim inf of a sequence and show that a sequence converges if and only if the lim sup equals the lim inf* – develop a sequence that converges to the square root – develop the sequence that converges to the Euler-Mascheroni constant	HW #12 Midterm Exam III Final Exam
– define a series in terms of partial sums of a sequence – distinguish between convergence and divergence of series – prove the Divergence Theorem and the Convergence Properties for series – analyze and apply convergence criteria for series with positive terms, including boundedness, comparison/limit comparison, and ratio tests	Midterm Exam III Final Exam

The table below indicates to what extent this course reflects each of the learning objectives of the undergraduate mathematics program. A description of learning objectives is available online at http://www.math.buffalo.edu/undergraduate/undergrad_programs.shtml.

Computational Skills:	**Analytical Skills:**	**Practical Problem Solving:**	**Research Skills:**	**Communication Skills:**
little or not at all	extensively	little or not at all	extensively	extensively