

Workshop

Functional Analysis and Operator Theory in Honor of Lewis A. Coburn

June 19, 2017
9:30h - 17:30h

Leibniz Universität Hannover
Welfengarten 1, 30167 Hannover
room C311

Lewis A. Coburn
Toeplitz quantization

Albrecht Böttcher
How to solve an equation with a Toeplitz operator?

Bernhard Gramsch
Division of distributions for Fredholm functions

Raffael Hagger
On the essential spectrum of Toeplitz operators

Ernst Albrecht
Small Möbius invariant spaces

Harald Upmeyer
Analysis and quantization of Kepler manifolds

Elmar Schrohe
Elliptic operators associated with groups of quantized
canonical transformations

Organizers:

Wolfram Bauer
Raffael Hagger
Elmar Schrohe

<https://www.math-conf.uni-hannover.de/lac>

Talks and Schedule

09:30 - 10:15	Lewis A. Coburn	“Toeplitz Quantization”
10:15 - 11:00	Albrecht Böttcher	“How to solve an equation with a Toeplitz operator?”
Coffee		
11:30 - 12:15	Bernhard Gramsch	“Division of distributions for Fredholm functions”
Lunch		
14:00 - 14:45	Raffael Hagger	“On the essential spectrum of Toeplitz operators”
14:45 - 15:30	Ernst Albrecht	“Small Möbius invariant spaces”
Coffee		
16:00 - 16:45	Harald Upmeyer	“Geometry and analysis on Kepler manifolds”
16:45 - 17:30	Elmar Schrohe	“Elliptic operators associated with groups of quantized canonical transformations”
18:30 -	Dinner at “Bei Mario”, Schloßstr. 6, 30159 Hannover	

Toeplitz quantization

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I discuss some recent work with Wolfram Bauer and Raffael Hagger. Here, \mathbb{C}^n is complex n -space and, for z in \mathbb{C}^n , we consider the standard family of Gaussian measures $d\mu_t(z) = (4\pi t)^{-n} \exp(-|z|^2/4t) dv(z)$, $t > 0$ where dv is Lebesgue measure. We consider the Hilbert space L_t^2 of all μ_t -square integrable complex-valued measurable functions on \mathbb{C}^n and the closed subspace of all square-integrable entire functions, H_t^2 . For f measurable and h in H_t^2 with fh in L_t^2 , we consider the Toeplitz operators $T_f^{(t)}h = P^{(t)}(fh)$ where $P^{(t)}$ is the orthogonal projection from L_t^2 onto H_t^2 . For bounded f (f in L^∞) and some unbounded f , these are bounded operators with norm $\|\cdot\|_t$. For f, g bounded, with “sufficiently many” bounded derivatives, there are known deformation quantization conditions, including

$$\lim_{t \rightarrow 0} \|T_f^{(t)}\|_t = \|f\|_\infty \quad \text{and} \quad (0)$$

$$\lim_{t \rightarrow 0} \|T_f^{(t)}T_g^{(t)} - T_{fg}^{(t)}\|_t = 0. \quad (1)$$

We exhibit bounded real-analytic functions F, G so that (1) fails. On the positive side, for the space VMO of functions with vanishing mean oscillation, we show that (1) holds for all f in (the sup-norm closed algebra) $\text{VMO} \cap L^\infty$ and g in L^∞ . (1) also holds for all f in UC (uniformly continuous functions, bounded or not) while (0) holds for all bounded continuous f .

How to solve an equation with a Toeplitz operator?

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An equation with a Hardy space Toeplitz operator can be solved by Wiener-Hopf factorization. However, Wiener-Hopf factorization does not work for Bergman space Toeplitz operators. The only way I see to tackle equations with a Toeplitz operator on the Bergman space is to have recourse to approximation methods. Several such methods are presented. I also embark on the higher-dimensional case, that is, on Toeplitz operators over the Bergman space of the polydisk and over the Segal-Bargmann space in several variables. I hope the talk is nice tour through some problems in the intersection of operator theory, complex analysis, differential geometry, and numerical mathematics.

Division of distributions for Fredholm functions

BERNHARD GRAMSCH

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The complex homotopy principle (cf., e.g. [1]) in the version of Bungart and Leiterer (cf.: [2]-[4]) is applied to the parameter dependent factorisation for distributions

$$D(t)T(t) = B(t), \quad (1)$$

where B and D are operator valued distributions and T an analytic Fredholm function depending continuously on a parameter t (cf.: [5]). Despite the classical counterexamples in the scalar valued case some sufficient conditions can be given for (1) with $T(t, z)$, z one complex variable, and in the case of “transformation parameters” t for several real or complex variables. In some cases the “quotient D ” of B and the multiplication with T has the form

$$D(t) = B(t)A(t) + S(t) \quad (2)$$

with a multiplication operator $A(t)$ and an operator distribution $S(t)$ with values in small ideals with fast decreasing approximation numbers. The methods developed lead to possibly new results for parameter dependent scalar and matrix valued distributions. These methods are based partially on recent joint work with W. Kabbalo. The theory also applies to some Fréchet operator algebras with spectral invariance, especially to some Hörmander classes of pseudodifferential operators. A series of problems is presented concerning (1) and (2) connected to lifting methods.

[1] B. Ivarsson, F. Kutzschebauch, *Holomorphic factorization of mappings into $SL_n(\mathbb{C})$* , Ann. of Math. 175 (2012), 45-69.

[2] B. Gramsch, *Oka's principle for special Fréchet Lie groups and homogeneous manifolds in topological algebras of the microlocal analysis*, in: Banach algebras 97, Proc. 13th Internat. Conf. on Banach Algebras (Blaubeuren, 1997), E. Albrecht and M. Mathieu, eds., de Gruyter, Berlin, Boston, 1998, 189-204.

[3] B. Gramsch, *Lifting of idempotent operator functions*, in: Banach algebras 97, Proc. 13th Internat. Conf. on Banach Algebras (Blaubeuren, 1997), E. Albrecht and M. Mathieu, eds., de Gruyter, Berlin, Boston, 1998, 527-533.

[4] B. Gramsch, *List of Open Problems*, in: Banach algebras 97, Proc. 13th Internat. Conf. on Banach Algebras (Blaubeuren, 1997), E. Albrecht and M. Mathieu, eds., de Gruyter, Berlin, Boston, 1998, 552-554.

[5] W. Kabbalo, *Meromorphic generalized inverses of operator functions*, Indag. Math. 23 (2012), 970-994.

On the essential spectrum of Toeplitz operators

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For $p \in (1, \infty)$ let $L^p(\mathbb{B}^n)$ denote the usual Lebesgue space over the unit ball and let $A^p(\mathbb{B}^n)$ denote the subspace of holomorphic functions. In this talk we introduce a certain type of bounded linear operators that we will call band-dominated operators. This notion originates from the theory of infinite matrices, where band-dominated operators appear as a limit of band matrices acting on the sequence space $\ell^p(\mathbb{Z})$. These operators have a very rich Fredholm theory and a lot of progress has been made in recent years (e.g. [1]). Our goal is to show similar results for Toeplitz operators on $A^p(\mathbb{B}^n)$. The key observation here is that Toeplitz operators are actually band-dominated if we extend them to all of $L^p(\mathbb{B}^n)$. This allows us to use similar methods as in the sequence space case and show equally strong results in terms of limit operators. Roughly speaking, limit operators are operators that appear when we shift our operator to the boundary $\partial\mathbb{B}^n$. As our main result we obtain that an operator in the Toeplitz algebra is Fredholm if and only if all of its limit operators are invertible. This extends a lot of well known results, e.g. that a Toeplitz operator T_f with $f \in C(\overline{\mathbb{B}^n})$ is Fredholm if and only if f has no zeros on the boundary $\partial\mathbb{B}^n$.

[1] M. Lindner, M. Seidel, *An affirmative answer to a core issue on limit operators*, J. Funct. Anal., Vol. 267, No. 3 (2014), 901-917.

Small Möbius invariant spaces

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In this survey we consider F -spaces of analytic functions on the unit ball \mathbb{B}_n in \mathbb{C}^n that are (generalized) Möbius invariant and are minimal with respect to certain weakened convexity conditions. We describe an elementary construction method to obtain such small spaces. Typical examples are the analytic Besov spaces $B^p(\mathbb{B}_n)$ for $0 < p \leq 1$ which are in addition topological algebras. We also consider some results which are special for the unit disc case $n = 1$. In particular, we give a characterization of all compact composition operators on $B^p(\mathbb{D})$ for all $p \in (0, 1]$.

Geometry and analysis on Kepler manifolds

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Kepler manifolds and varieties generalize the well-known determinantal varieties (vanishing of minors) in matrix spaces. The simplest case is the complexified light cone $z_1^2 + \dots + z_n^2 = 0$ in \mathbb{C}^n . In the talk we describe the geometry and analysis of Kepler manifolds, defined in general using Jordan algebras and Jordan triples. The principal results, in joint work with M. Englis, are

- Kepler varieties are normal varieties, allowing a Riemann extension theorem for holomorphic functions,
- Kepler manifolds have invariant holomorphic n -forms and measures, which will be described explicitly,
- The associated Hilbert spaces of holomorphic functions have a reproducing kernel function (generalized Bergman kernel) with an explicit Peter-Weyl decomposition over all partitions,
- For suitable plurisubharmonic functions, the reproducing kernel has an asymptotic (Tian-Yau-Zelditch) expansion, using a (multi-variable) confluent hypergeometric function $F_{1,1}$.

Some applications to Hankel operators are also discussed.

Elliptic operators associated with groups of quantized canonical transformations

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Given a Lie group G of quantized canonical transformations acting on the space $L^2(M)$ over a closed manifold M , we define an algebra of so-called G -operators on $L^2(M)$. We show that to G -operators we can associate symbols in appropriate crossed products with G , introduce a notion of ellipticity and prove the Fredholm property for elliptic elements.

This framework encompasses many known elliptic theories, for instance, shift operators associated with group actions on M , transversal elliptic theory, transversally elliptic pseudodifferential operators on foliations, and Fourier integral operators associated with coisotropic submanifolds.

Joint work with A. Savin and B. Sternin.