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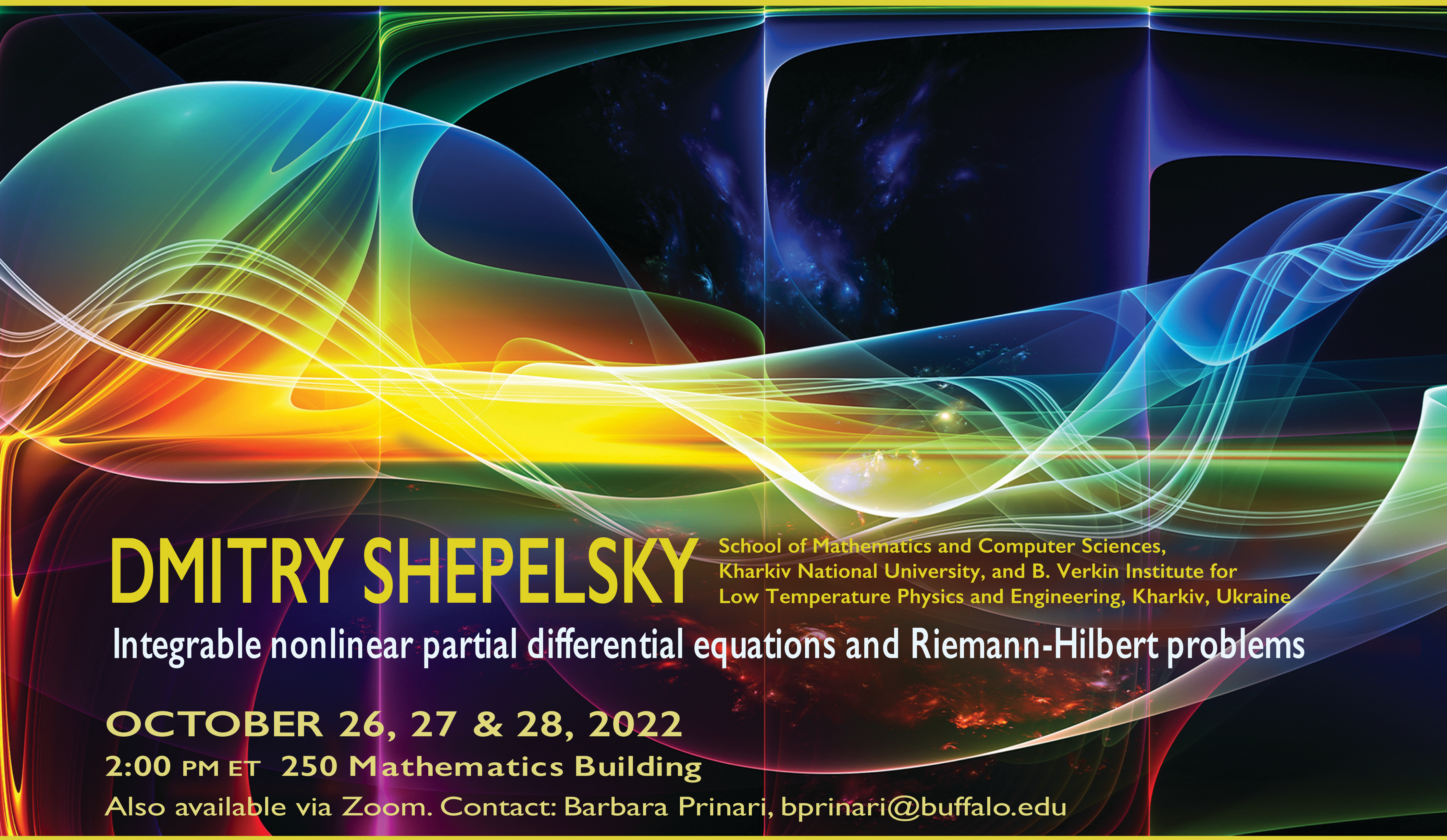

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Integrable nonlinear partial differential equations and Riemann-Hilbert problems

OCTOBER 26, 27 & 28, 2022

2:00 PM ET 250 Mathematics Building

Also available via Zoom. Contact: Barbara Prinari, bprinari@buffalo.edu

In contrast with linear partial differential equations (PDE), there is no general theory of nonlinear partial differential equations. But for some classes of nonlinear PDE, it turns out that one can not only construct a vast amount of exact solutions but also solve, in a quite efficient way, general problems of certain type, like the Cauchy (initial value) problems. This holds for a class of nonlinear PDE called integrable equations. Also known as soliton equations, they support solutions (solitons) which are localized, stable nonlinear travelling waves that retain their shape and speed after interactions.

LECTURE 1

In this lecture, I introduce a basic property which characterizes a PDE as being integrable — the Lax pair representation, and discuss how to use an analogous property of linear PDE in order to obtain a representation of the solution of the Cauchy problem (for a linear PDE that can be viewed as a linearization of an integrable PDE) in terms of the solution of an associated Riemann-Hilbert problem. For this, I discuss some basics of Riemann-Hilbert problems, which are boundary-value problems for (piece-wise) analytic functions in the complex plane.

LECTURE 2

In this lecture, we discuss how to obtain a representation of the solution of the Cauchy problem for a nonlinear PDE in terms of the solution of an appropriate Riemann-Hilbert problem. As a prototype model, we use the nonlinear Schrödinger equation, which is a basic model of nonlinear wave propagation in various situations. In particular, we discuss the soliton solutions in the framework of the RH problem approach. We also discuss the notions of regular and irregular Riemann-Hilbert problem and how they can be related. Finally, we touch upon the ideas behind the nonlinear steepest descent method for studying the long-time behavior of solutions of the Cauchy problems.

LECTURE 3

In this lecture, we discuss the adaptation of the Riemann-Hilbert approach to a problem with non-trivial (non-zero) boundary conditions and the subsequent generalization of the nonlinear steepest descent method for studying the long-time asymptotics. The latter necessitates the introduction of the so-called g-function mechanism, whose basic principles will be also discussed and illustrated in the case of the nonlinear Schrödinger equation.

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