Second Qualifying Exam Information

ANALYSIS

A. Real Analysis

1. Semi-continuous functions. Measures, σ-algebras, measurable sets and functions, Borel sets, measure spaces. Lebesgue measure and integration, Lusin's theorem, Egoroff's theorem, Vitali-Caratheodory theorem.
2. Lp spaces, bounded linear functionals on Lp. Elementary Hilbert space theory, subspaces, representation theorems, orthonormal systems. Elementary Banach space theory including Baire's theorem, uniform boundedness principle, open mapping theorem, Hahn-Banach theorem.
3. Radon-Nikodym Theorem. Product measures, Fubini's Theorem. Functions of bounded variation and absolutely continuous functions.

References:

1. Rudin, Real and Complex Analysis (Chapters 1-8)
2. Royden, Real Analysis

B. Complex Analysis

1. Complex numbers, analytic functions, Cauchy Riemann equations, relation between harmonic and analytic functions.
2. Complex integration, Cauchy integral theorem and formulas, Morera's theorem, Liouville's Theorem, maximum modulus principle.
3. Power series and Laurent series representations of analytic functions. Zeros, Isolated singularities. Identity theorem.
4. Residue theorem, evaluation of definite integrals, argument principle.
5. Entire functions. Casorati-Weierstrass theorem.
6. Conformal mapping, linear fractional transformations.

References:

1. J.B. Conway, *Functions of One Complex Variable* (Chapters I-VI)
2. L.V. Ahlfors, *Complex Analysis* (Chapters I-V)
3. Rudin, *Real and Complex Analysis* (Chapters 10-14)

ALGEBRA

1. **Groups**. symmetry groups, homomorphism theorems, Sylow theorems, group actions on sets.
2. **Rings**. various examples (*e.g.*, rings of continuous or analytic functions), unique factorization domains, Gauss' lemma, Eisenstein criterion, Noetherian rings, Artinian rings, Semi-simple rings, Wedderburn-Artin theorems, group rings, Maschke's theorem.
3. **Modules**. tensor products, exterior powers, projective and injective modules, Nakayama's lemma, modules over principal ideal domains, modules over semi-simple rings, group representations Jordan and rational canonical forms, Cayley-Hamilton theorem, determinants.

4. **Fields**. field extensions, finite multiplicative subgroups of a field, structure of finite fields, irreducibility of the cyclotomic polynomials, Galois theory, algebraic closure, transcendental extensions.
5. **Category theory**. representable functors, adjoint functors, universal properties, Yoneda's lemma.

References:

1. E. Artin, *Galois Theory*
2. S. Lang, *Algebra*
3. N. Jacobson, *Basic Algebra I and II*

GEOMETRY/TOPOLOGY

A. Algebraic Topology

1. Homotopy, fundamental group, covering spaces, Van Kampen's theorem
2. Simplicial and cell complexes, singular homology and cohomology groups
3. The exact homology sequence, the excision theorem, Mayer-Vietoris sequence, Jordan-Brouwer separation theorem
4. Statements and applications of the Künneth theorem and the Universal Coefficient theorem
5. Orientation of manifolds, cup product, Poincaré-Lefschetz duality

Suggested References:

1. Hatcher, *Algebraic Topology*
2. Bredon, *Topology and Geometry* [Chapters 3, 4, 6]

B. Differential Geometry

1. Manifolds, implicit and inverse function theorems
2. Submersions, immersions, embeddings and tranversality
3. Regular values, critical values and Sard's theorem
4. Differential forms, Stokes' theorem, de Rham cohomology

Suggested References:

1. Guillemin and Pollack, *Differential Topology*
2. Bredon, *Topology and Geometry* [Chapters 2, 5]

DIFFERENTIAL EQUATIONS

A. Ordinary Differential Equations

1. Existence and uniqueness of solutions to initial value problems for single equations and systems
2. Solution of linear first order systems, especially constant coefficient systems

3. Qualitative analysis for nonlinear systems, phase portraits, classification of equilibrium states, Poincaré-Bendixson theorem, Lyapunov functions, Lienard and van der Pol equations
4. Floquet theory and the stability of periodic solutions, stable manifold theorem, invariant manifolds
5. Sturm-Liouville and two-point boundary value problems

Suggested References:

1. Hale, *Ordinary Differential Equations*
2. Hirsch and Smale, *Differential Equations, Dynamical Systems and Linear Algebra*
3. Perko, *Differential Equations and Dynamical Systems*

B. Partial Differential Equations

1. Linear and non-linear equations of first order, characteristics, Hamilton-Jacobi equations, equations of geometrical optics
2. Classification of PDE
3. Fundamental solutions of elliptic and parabolic equations, especially the Laplace, Helmholtz and heat equations
4. Dirichlet and Neumann problems for Laplace, Helmholtz and heat equations, maximum principle and uniqueness theorems for elliptic and parabolic equations
5. Solution of the initial value problem for the wave equation, conservation of energy and uniqueness theorems for the wave equation, Huyghen's principle
6. Fredholm Alternative and eigenfunction expansion with applications to elliptic, parabolic and hyperbolic equations

Suggested References:

1. Evans, *Partial Differential Equations*
2. John, *Partial Differential Equations*
3. Guenther and Lee, *Partial Differential Equations of Mathematical Physics and Integral Equations*